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## Chapter 1

### INTRODUCTION TO TURBOMACHINES

#### 1.1 Introduction:

The turbomachine is used in several applications, the primary ones being electrical power generation, aircraft propulsion and vehicular propulsion for civilian and military use. The units used in power generation are steam, gas and hydraulic turbines, ranging in capacity from a few kilowatts to several hundred and even thousands of megawatts, depending on the application. Here, the turbomachines drives the alternator at the appropriate speed to produce power of the right frequency. In aircraft and heavy vehicular propulsion for military use, the primary driving element has been the gas turbine.

#### 1.2 Turbomachines and its Principal Components:

**Question No 1.1:** Define a turbomachine. With a neat sketch explain the parts of a turbomachine. (VTU, Dec-06/Jan-07, Dec-12, Dec-13/Jan-14)

**Answer:** A turbomachine is a device in which energy transfer takes place between a flowing fluid and a rotating element due to the dynamic action, and results in the change of pressure and momentum of the fluid.

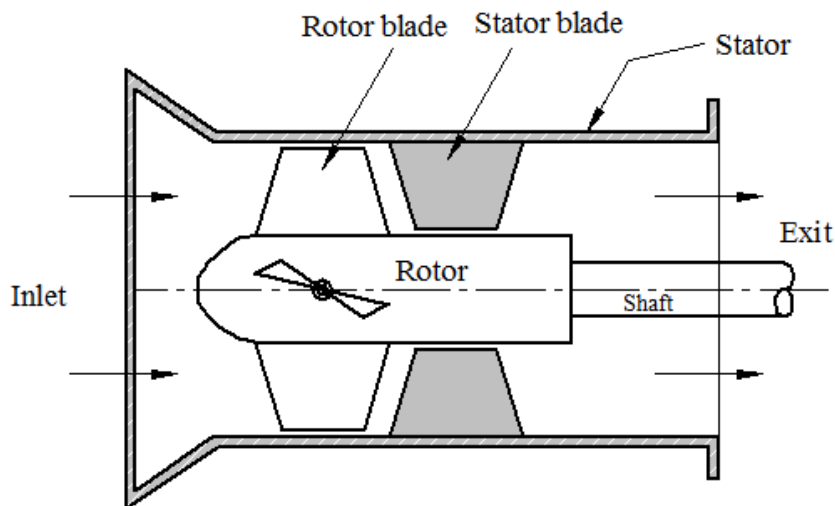


Fig. 1.1 Principal components of turbomachine

The following are the principal components of turbomachine: (i) Rotor, (ii) Stator and (iii) Shaft.

Rotor is a rotating element carrying the rotor blades or vanes. Rotor is also known by the names runner, impellers etc. depending upon the particular machine. Here energy transfer occurs between the flowing fluid and the rotating element due to the momentum exchange between the two.

Stator is a stationary element carrying the guide vanes or stator blades. Stator blades are also known by guide blades or nozzle depending upon the particular machine. These blades usually control the direction of fluid flow during the energy conversion process.

Shaft is transmitting power into or out of the machine depending upon the particular machine. For power generating machines, it may call as output shaft and for power absorbing machines; it may called as input shaft.

### 1.3 Classification of Turbomachines:

**Question No 1.2:** Explain how turbomachines are classified. Give at least one example of each. (VTU, Feb-06, Jun/Jul 14)

**Answer:** Turbomachines are broadly classified into power generating, power absorbing and power transmitting turbomachines.

In power-generating turbomachines, fluid energy (decrease in enthalpy) is converted into mechanical energy which is obtained at the shaft output, whereas in power-absorbing turbomachines, mechanical energy which is supplied at the shaft input is converted to fluid energy (increase in enthalpy). The power-transmitting turbomachines are simply transmitting power from input shaft to an output shaft. That means, these devices act merely as an energy transmitter to change the speed and torque on the driven member as compared with the driver.

Again power-generating and power-absorbing turbomachines are classified by the direction of the fluid flow as: (i) axial flow, (ii) radial flow and (iii) mixed flow. In the axial flow and radial flow turbomachines, the major flow directions are approximately axial and radial respectively, while in the mixed flow machine, the flow enters axially and leaves radially or vice versa. A radial flow machine may also be classified into radial inward flow (centripetal) or radial outward flow (centrifugal) types depending on whether the flow is directed towards or away from the shaft axis.

**Question No 1.3:** Explain with examples the power generating, power absorbing and power transmitting turbomachines. (VTU, Jul/Aug-02, Jun/Jul-13)

**Answer:** Power generating turbomachine is one which converts fluid energy in the form of kinetic energy or pressure energy into mechanical energy in terms of rotating shaft. Turbines are the best example for this type.

Power absorbing turbomachine is one which converts mechanical energy into fluid energy. Compressors, fans, pumps and blowers are the best example for this type.

Power transmitting is one which is used to transmit power from driving shaft to driven shaft with the help of fluid. There is no mechanical connection between the two shafts. The best examples for this type are hydraulic coupling and hydraulic torque converter.

**Question No 1.4:** What is an axial flow turbomachine? How is it different from a radial flow turbomachine? Give one example each.

**Answer:** In axial flow turbomachine, the major flow direction is approximately axial, example: Kaplan turbine. Whereas in radial flow turbomachine, the major flow direction is radial, example: Francis turbine.

#### 1.4 Positive-Displacement Devices and Turbomachines:

**Question No 1.5:** Compare the turbomachines with positive displacement machines. (VTU, Jan/Feb-02, Jan/Feb-03, Jan/Feb-04, Dec-12, Jun/Jul-13)

**Answer:** The differences between positive-displacement machines and turbomachines are given by comparing their modes of action, operation, energy transfer, mechanical features etc. in the following table.

Modes	Positive-displacement Machine	Turbomachine
<b>Action</b>	(a) It creates thermodynamic and mechanical action between a nearly static fluid and a relatively slowly moving surface.	(a) It creates thermodynamic and dynamic interaction between a flowing fluid and rotating element.
	(b) It involves a change in volume or a displacement of fluid.	(b) It involves change in pressure and momentum of the fluid.
	(c) There is a positive confinement of the fluid in the system.	(c) There is no positive confinement of the fluid at any point in the system.
<b>Operation</b>	(a) It involves a reciprocating motion of the mechanical element and unsteady flow of the fluid. But some rotary positive displacement machines are also built. Examples: Gear pump, vane pump	(a) It involves a purely rotary motion of mechanical element and steady flow of the fluid. It may also involve unsteady flow for short periods of time, especially while starting, stopping or during changes of load.
	(b) Entrapped fluid state is different from the surroundings when the machine is stopped, if heat transfer and leakages are avoided.	(b) The fluid state will be the same as that of the surroundings when the machine is stopped.
<b>Mechanical</b>	(a) Because of the reciprocating masses,	(a) Rotating masses can be completely

<b>Features</b>	vibrations are more. Hence low speeds are adopted.	balanced and vibrations eliminated. Hence high speeds can be adopted.
	(b) Heavy foundations are required.	(b) Light foundations sufficient.
	(c) Mechanical design is complex because of valves.	(c) Design is simple.
	(d) Weight per unit output is more.	(d) Weight per unit output is less.
<b>Efficiency of conversion process</b>	(a) High efficiency because of static energy transfer.	(a) Efficiency is low because of dynamic energy transfer.
	(b) The efficiencies of the compression and expansion processes are almost the same.	(b) The efficiency of the compression process is low.
<b>Volumetric efficiency</b>	(a) Much below that of a turbomachine because of valves.	(a) It is almost 100%.
	(b) Low fluid handling capacity per unit weight of machine.	(b) High fluid handling capacity per unit weight of machine.
<b>Fluid phase change and surging</b>	No such serious problems are encountered.	(a) Causes cavitation in pumps and turbines. Therefore leads to erosion of blades.
		(b) Surging or pulsation leads to unstable flow. And also causes vibrations and may destroy the machine.
		(c) These factors deteriorate the performance of the machine.

**Question No 1.6: Are vane compressors and gear pumps turbomachines? Why? (VTU, Dec-10)**

**Answer:** No, vane compressors and gear pumps are positive displacement machines and work by moving a fluid trapped in a specified volume (i.e., fluid confinement is positive).

### **1.5 First and Second Laws of Thermodynamics Applied to Turbomachines:**

**Question No 1.7: Explain the applications of first and second laws of thermodynamics to turbomachines. (VTU, Jul/Aug-02) Or,**

**Starting from the first law, derive an expression for the work output of a turbomachine in terms of properties at inlet and outlet. Or,**

**Deducing an expression, explain the significance of first and second law of thermodynamics applied to a turbomachine. (VTU, Dec-12, Dec 14/Jan 15)**

**Answer:** Consider single inlet and single output steady state turbomachine, across the sections of which the velocities, pressures, temperatures and other relevant properties are uniform.

**Application of first law of thermodynamics:** The steady flow equation of the first law of thermodynamics in the unit mass basis is:

$$q + h_1 + \frac{v_1^2}{2} + gz_1 = w + h_2 + \frac{v_2^2}{2} + gz_2 \quad (1.1)$$

Here,  $q$  and  $w$  are heat transfer and work transfer per unit mass flow across the boundary of the control volume respectively.

Since, the stagnation enthalpy:  $h_o = h + \frac{v^2}{2} + gz$ .

Then, equation (1.1) becomes:  $q - w = h_{o2} - h_{o1} = \Delta h_o$  (1.2)

Generally, all turbomachines are well-insulated devices, therefore  $q=0$ . Then equation (1.2) can be written as:  $\Delta h_o = -w$  (1.3)

The equation (1.3) represents that, *the energy transfer as work is numerically equal to the change in stagnation enthalpy of the fluid between the inlet and outlet of the turbomachine.*

In a power-generating turbomachine,  $w$  is positive as defined so that  $\Delta h_o$  is negative, i.e., the stagnation enthalpy at the exit of the machine is less than that at the inlet. The machine produces out work at the shaft. In a power-absorbing turbomachine,  $w$  is negative as defined so that  $\Delta h_o$  is positive. The stagnation enthalpy at the outlet will be greater than that at the inlet and work is done on the flowing fluid due to the rotation of the shaft.

**Application of second law of thermodynamics:** The second law equation of states, applied to stagnation properties is:

$$T_o ds_o = dh_o - v_o dp_o \quad (1.4)$$

But equation (1.3) in differential form is,  $dh = -dw$ .

Then equation (1.4) can be written as:

$$-dw = v_o dp_o + T_o ds_o \quad (1.5)$$

In a power-generating machine,  $dp_o$  is negative since the flowing fluid undergoes a pressure drop when mechanical energy output is obtained. However, the Clausius inequality for a turbomachine is given that  $T_o ds_o \geq 0$ . The sign of equality applies only to a reversible process which has a work output  $dw_{rev} = v_o dp_o$ . In a real machine (irreversible machine),  $T_o ds_o > 0$ , which has a work output  $dw_{irr} = v_o dp_o - T_o ds_o$ . So that  $dw_{rev} - dw_{irr} = T_o ds_o$  and represents the decrease in work output due to the irreversibilities in the machine. Therefore the reversible power-generating machine exhibits the highest mechanical output of all the machines undergoing a given stagnation pressure

change. A similar argument may be used to prove that the reversible power-absorbing machine needs the minimum work input of all the machines for a given stagnation pressure rise (i.e.,  $dw_{irr} - dw_{rev} = T_o ds_o$ ).

## 1.6 Efficiency of Turbomachines:

**Question No 1.8: Define: (i) adiabatic efficiency and (ii) mechanical efficiency for power generating and power absorbing turbomachines. (VTU, Dec-12)**

**Answer:** The performance of a real machine is always inferior to that of a frictionless and loss-free ideal machine. A measure of its performance is the efficiency, defined differently for power-generating and power-absorbing machines.

For power-generating machine, the efficiency is defined as:

$$\eta_{pg} = \frac{\text{Actual Shaft Work Output}}{\text{Ideal Work Output}} = \frac{w_{sft}}{w_i}$$

$$\text{Or, } \eta_{pg} = \frac{\text{Actual Shaft Work Output}}{\text{Hydrodynamic Energy Available from the Fluid}} = \frac{w_{sft}}{w_i}$$

For power-absorbing machine, the efficiency is defined as:

$$\eta_{pa} = \frac{\text{Ideal Work Input}}{\text{Actual Shaft Work Input}} = \frac{w_i}{w_{sft}}$$

$$\text{Or, } \eta_{pa} = \frac{\text{Hydrodynamic Energy Supplied to the Fluid}}{\text{Actual Shaft Work Input}} = \frac{w_i}{w_{sft}}$$

Generally, losses occur in turbomachines are due to: (a) mechanical losses like bearing friction, windage, etc., (b) fluid-rotor losses like unsteady flow, friction between the blade and the fluid, leakage across blades etc. If the mechanical and fluid-rotor losses are separated, the efficiencies may be rewritten in the following forms:

For power-generating turbomachine,

$$\eta_{pg} = \frac{\text{Mechanical Energy Supplied by the Rotor}}{\text{Hydrodynamic Energy Available from the Fluid}} \times \frac{\text{Actual Shaft Work Output}}{\text{Mechanical Energy Supplied by the Rotor}}$$

$$\text{Or, } \eta_{pg} = \eta_a \times \eta_m$$

For power-absorbing turbomachines,

$$\eta_{pa} = \frac{\text{Hydrodynamic Energy Supplied to the Fluid}}{\text{Mechanical Energy Supplied to the Rotor}} \times \frac{\text{Mechanical Energy Supplied to the Rotor}}{\text{Actual Shaft Work Input}}$$

$$\text{Or, } \eta_{pg} = \eta_a \times \eta_m$$

where  $\eta_a$  and  $\eta_m$  are adiabatic and mechanical efficiencies respectively.

For power-generating turbomachine, adiabatic or isentropic or hydraulic efficiency may be written as,



$$\eta_a = \frac{\text{Mechanical Energy Supplied by the Rotor}}{\text{Hydrodynamic Energy Available from the Fluid}} = \frac{w_r}{w_i}$$

For power-absorbing turbomachine, adiabatic or isentropic or hydraulic efficiency may be written as,

$$\eta_a = \frac{\text{Hydrodynamic Energy Supplied to the Fluid}}{\text{Mechanical Energy Supplied to the Rotor}} = \frac{w_i}{w_r}$$

Note: (i) Hydrodynamic energy is defined as the energy possessed by the fluid in motion.

(ii) Windage loss is caused by fluid friction as the turbine wheel and blades rotate through the surrounding fluid.

(iii) Leakage loss is caused by the fluid when it passes over the blades tip without doing any useful work.

## 1.7 Dimensional Analysis:

The dimensional analysis is a mathematical technique deals with the dimensions of the quantities involved in the process. Basically, dimensional analysis is a method for reducing the number and complexity of experimental variable that affect a given physical phenomenon, by using a sort of compacting technique.

The three primary purposes of dimensional analysis are:

1. To generate non-dimensional parameters that help in the design of experiments and in the reporting of experimental results.
2. To obtain scaling laws so that prototype performance can be predicted from model performance.
3. To predict the relationship between the parameters.

**1.7.1 Fundamental Quantities:** Mass (M), length (L), time (T) and temperature ( $\theta$ ) are called fundamental quantities since there is no direct relation between these quantities. There are seven basic quantities in physics namely, mass, length, time, electric current, temperature, luminous intensity and amount of a substance.

**1.7.2 Secondary Quantities or Derived Quantities:** The quantities derived from fundamental quantities are called derived quantities or secondary quantities. Examples: area, volume, velocity, force, acceleration, etc.

**1.7.3 Dimensional Homogeneity:** An equation is said to be dimensionally homogeneous if the fundamental dimensions have identical powers of M, L, T on both sides.

For example:

$$Q = AV$$

In dimensional form:

$$\frac{L^3}{T} = L^2 \times \frac{L}{T} = \frac{L^3}{T}$$

## 1.8 Buckingham's $\pi$ -Theorem:

The Buckingham's  $\pi$ -theorem states that "if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions such as M, L, T then they may be grouped into (n-m), non-dimensional independent  $\pi$ -terms".

Let a variable  $X_1$  depends upon independent variables  $X_2, X_3, \dots, X_n$ . The functional equation may be written as:

$$X_1 = f(X_2, X_3, \dots, X_n) \quad (1.6)$$

The above equation can also be written as:

$$f(X_1, X_2, X_3, \dots, X_n) = C \quad (1.7)$$

Where, C is constant and f is some function.

In the above equation (1.7), there are 'n' variables. If these variables contain 'm' fundamental dimensions, then according to Buckingham's  $\pi$ -theorem,

$$f_1(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = C \quad (1.8)$$

### 1.9 Procedure for Applying Buckingham's $\pi$ -Theorem:

- 1) With a given data, write the functional relationship.
- 2) Write the equation in its general form.
- 3) Choose repeating variables and write separate expressions for each  $\pi$ -term, every  $\pi$ -term must contain the repeating variables and one of the remaining variables. In selecting the repeating variable the following points must be considered:
  - (a) Never pick the dependent variable.
  - (b) The chosen repeating variables must not by themselves be able to form a dimensionless group. Example: V, L and t are not considered as a repeating variable, because  $\frac{Vt}{L}$  will be a non-dimensional.
  - (c) The chosen repeating variables must represent all the primary dimensions in the problem.
  - (d) Never pick the variables that are already dimensionless. These are  $\pi$ 's already, all by themselves.
  - (e) Never pick two variables with the same dimensions or with dimensions that differ by only an exponent. That is one variable contains geometric property, second variable contains flow property and third containing fluid property.
  - (f) Pick simple variables over complex variables whenever possible.
  - (g) Pick popular parameters since they may appear in each of the  $\pi$ 's.
- 4) The repeating variables are written in exponential form.
- 5) With the help of dimensional homogeneity, find the values of exponents by obtaining simultaneous equations.

6) Now, substitute the values of these exponents in the  $\pi$  terms.

7) Write the functional relation in the required form.

**1.8.1 Geometric Variables:** The variables with geometric property in turbomachines are *length, diameter, thickness, height* etc.

**1.8.2 Kinematic Variables:** The variables with flow property in turbomachines are *velocity, speed, volume flow rate, acceleration, angular velocity* etc.

**1.8.3 Dynamic Variables:** The variables with fluid property in turbomachines are *mass flow rate, gas density, dynamic viscosity, bulk modulus, pressure difference, force, power, elasticity, surface tension, specific weight, stress, resistance* etc.

**Note:** (1) For power generating turbomachines, the performance of a machine is referred to the power developed (P), workdone (W), pressure ratio ( $P_1/P_2$ ) or efficiency ( $\eta$ ) which depend on independent variables.

(2) For power absorbing turbomachines, the performance is referred to the discharge (Q), enthalpy rise ( $\Delta h$ ), pressure ratio ( $P_2/P_1$ ) or efficiency ( $\eta$ ) which depend on independent variables.

**Question No 1.9:** Performance of a turbomachine depends on the variables discharge (Q), speed (N), rotar diameter (D), energy per unit mass flow (gH), power (P), density of fluid ( $\rho$ ), dynamic viscosity of fluid ( $\mu$ ). Using the dimensional analysis obtain the  $\pi$ -terms. (VTU, Jul/Aug-02)

**Answer:** General relationship is:

$$f(Q, N, D, gH, P, \rho, \mu) = \text{constant}$$

Dimensions:  $Q = L^3T^{-1}, N = T^{-1}, D = L, gH = L^2T^{-2}, P = ML^2T^{-3}, \rho = ML^{-3}, \mu = ML^{-1}T^{-1}$

Number of variables,  $n = 7$

Number of fundamental variables,  $m = 3$

Number of  $\pi$ -terms required,  $(n-m) = 4$

Repeating variables are:  $D, N, \rho$

$$\pi_1\text{-term: } \pi_1 = D^a N^b \rho^c Q$$

In dimensional form:  $M^0 L^0 T^0 = L^a (T^{-1})^b (ML^{-3})^c L^3 T^{-1}$

Equating the powers of M L T on both sides:

For M,  $0 = c$

For L,  $0 = a - 3c + 3 \Rightarrow a = -3$

For T,  $0 = -b - 1 \Rightarrow b = -1$

Then,  $\pi_1 = D^{-3} N^{-1} \rho^0 Q$

$$\pi_1 = \frac{Q}{ND^3}$$

$\pi_2$ -term:  $\pi_2 = D^a N^b \rho^c gH$

In dimensional form:  $M^0 L^0 T^0 = L^a (T^{-1})^b (ML^{-3})^c L^2 T^{-2}$

Equating the powers of M L T on both sides:

For M,  $0 = c$

For L,  $0 = a - 3c + 2 \Rightarrow a = -2$

For T,  $0 = -b - 2 \Rightarrow b = -2$

Then,  $\pi_1 = D^{-2} N^{-2} \rho^0 gH$

$$\pi_2 = \frac{gH}{N^2 D^2}$$

$\pi_3$ -term:  $\pi_3 = D^a N^b \rho^c P$

In dimensional form:  $M^0 L^0 T^0 = L^a (T^{-1})^b (ML^{-3})^c ML^2 T^{-3}$

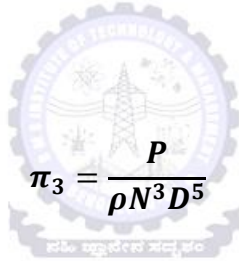
Equating the powers of M L T on both sides:

For M,  $0 = c + 1 \Rightarrow c = -1$

For L,  $0 = a - 3c + 2 \Rightarrow a = -5$

For T,  $0 = -b - 3 \Rightarrow b = -3$

Then,  $\pi_3 = D^{-5} N^{-3} \rho^{-1} P$



$$\pi_3 = \frac{P}{\rho N^3 D^5}$$

$\pi_4$ -term:  $\pi_4 = D^a N^b \rho^c \mu$

In dimensional form:  $M^0 L^0 T^0 = L^a (T^{-1})^b (ML^{-3})^c ML^{-1} T^{-1}$

Equating the powers of M L T on both sides:

For M,  $0 = c + 1 \Rightarrow c = -1$

For L,  $0 = a - 3c - 1 \Rightarrow a = -2$

For T,  $0 = -b - 1 \Rightarrow b = -1$

Then,  $\pi_4 = D^{-2} N^{-1} \rho^{-1} \mu$

$$\pi_4 = \frac{\mu}{\rho N D^2}$$

**Question No 1.10:** Give the significance of the dimensionless terms (i) Flow coefficient (ii) Head coefficient (iii) Power coefficient with respect to turbomachines. (VTU, Dec-06/Jan-07) Or, Explain capacity coefficient, head coefficient and power coefficient referring to a turbomachines. (VTU, Jan/Feb-02, Jan/Feb-03, Jan/Feb-04)

**Answer:** The various  $\pi$ -terms have the very significant role in a turbomachine as explained below.

**(i) Flow Coefficient:** It is also called as capacity coefficient or specific capacity. The term  $\frac{Q}{ND^3}$  is the capacity coefficient, which signifies the volume flow rate of fluid through a turbomachine of unit

diameter of runner operating at unit speed. The specific capacity is constant for dynamically similar conditions. Hence for a fan or pump of certain diameter running at various speeds, the discharge is proportional to the speed. This is the *First fan law*.

**Speed ratio:** The specific capacity is related to another quantity called speed ratio and is obtained as follows:

$$\frac{Q}{ND^3} \propto \frac{D^2V}{ND^3} \propto \frac{V}{ND} \propto \frac{V}{U} = \frac{1}{\phi} \quad (\text{Because } Q = AV = \frac{\pi D^2 V}{4} \propto D^2 V \text{ and also } U \propto ND)$$

Where  $\phi = \frac{U}{V}$  is called the speed ratio, which is defined as the ratio of tangential velocity of runner to the theoretical jet velocity of fluid. For the given machine, the speed ratio is fixed.

**(ii) Head Coefficient:** The term  $\frac{gH}{N^2 D^2}$  is called the head coefficient or specific head. It is a measure of the ratio of the fluid potential energy (column height H) and the fluid kinetic energy while moving at the rotational speed of the wheel U. The term can be interpreted by noting that:  $\frac{gH}{N^2 D^2} \propto \frac{gH}{U^2}$

The head coefficient is constant for dynamically similar machines. For a machine of specified diameter, the head varies directly as the square of the tangential speed of wheel. This is the Second fan law.

**(iii) Power Coefficient:** The term  $\frac{P}{\rho N^3 D^5}$  is called the power coefficient or specific power. It represents the relation between the power, fluid density, speed and wheel diameter. For a given machine, the power is directly proportional to the cube of the tangential speed of wheel. This is the Third fan law.

**Question No 1.11: Discuss the effect of Reynolds number on turbomachine. (VTU, Jun/Jul-08)**

**Answer:** The Reynolds number defined as the ratio of the inertial force to the viscous force. It is an important parameter, which represents the nature of flow. If the Reynolds number is greater than 4000, the flow is termed as turbulent, in which the inertia effect is more than the viscous effects. And, if Reynolds number is less than 2000, then flow is laminar in which viscous effects are more than the inertia effect.

The values of Reynolds number in turbines are much higher than the critical values. Most of the turbines use relatively low viscosity fluids like air, water and light oil. Therefore, the Reynolds number has very little effect on the power output of the machine. But, Reynolds number is an important parameter for small pumps, compressors, fans and blowers. Their performance improves with an increase in Reynolds number.

$$\text{The Reynolds number for the pipe flow is expressed as } R_e = \frac{\rho V D}{\mu}$$

### 1.10 Specific Speed:

The specific speed is the dimensionless term and is the parameter of greatest importance in incompressible flow machines. The specific speed is only the parameter that doesn't contain the linear

dimension of the runner. Hence, while operating under the same conditions of flow and head, all geometrically similar machines have the same specific speed, irrespective of their sizes.

The specific speed can be expressed in terms of discharge (Q) for power absorbing machine or the power (P) for power generating machine.

### 1.10.1 Specific Speed of a Pump:

**Question No 1.12:** Define specific speed of a pump. Derive an expression for specific speed of a pump from fundamentals. (VTU, Jul/Aug-05, Dec 14/Jan 15)

**Answer:** Specific speed can be defined as “a speed of geometrically similar machines discharging one cubic meter per second of water under head of one meter”.

Head coefficient is given by

$$\frac{gH}{N^2 D^2}$$

$$N^2 D^2 \propto gH$$

or

$$D \propto \frac{(gH)^{1/2}}{N} \quad (1.9)$$

Flow coefficient is given by

$$\frac{Q}{ND^3}$$

or

$$Q \propto ND^3$$

From equation (1.9)

$$Q \propto \frac{(gH)^{3/2}}{N^2}$$

or

$$Q = C \frac{(gH)^{3/2}}{N^2} \quad (1.10)$$

Where C is proportionality constant, from the definition of specific speed of pump:

$$\text{If } Q = 1 \text{ m}^3/\text{s}, \text{ and } H = 1\text{m}, \text{ then } N = N_s$$

Then equation (1.10) can be written as,  $C = \frac{N_s^2}{g^{3/2}}$  (1.11)

Substitute equation (1.11) in equation (1.10), then  $N_s = \frac{NQ^{1/2}}{H^{3/4}}$  (1.12)

The equation (1.12) gives the specific speed of a pump.

### 1.10.2 Specific Speed of a Turbine:

**Question No 1.13:** Define specific speed of a turbine. Obtain an expression for the same in terms of shaft power, speed and head. (VTU, Jun/Jul-08, Jun/Jul-13, Dec 14/ Jan 15)

**Answer:** Specific speed of a turbine is defined as “a speed of a geometrically similar machine which produces one kilowatt power under a head of one meter”.

Power coefficient is given by

$$\frac{P}{\rho N^3 D^5} \quad (1.13)$$

From equation (1.9)  $D \propto \frac{(gH)^{1/2}}{N}$ , then equation (1.13) can be written as,  $P \propto \frac{\rho(gH)^{5/2}}{N^2}$

or

$$P = C \frac{\rho(gH)^{5/2}}{N^2} \quad (1.14)$$

Where C is proportionality constant, from the definition of specific speed of turbine:

If  $P = 1kW$  and  $H = 1m$ , then  $N = N_s$

Then, equation (1.14) becomes  $C = \frac{N_s^2}{\rho g^{5/2}}$  (1.15)

Substitute equation (1.15) in equation (1.14), then  $N_s = \frac{NP^{1/2}}{H^{5/4}}$  (1.16)

The equation (1.16) gives the specific speed of a turbine.

### 1.10.3 Significance of Specific Speed:

**Question No 1.14:** Briefly explain the significance of specific speed related to turbomachines.

(VTU, Jul-06, Dec-13/Jan-14)

**Answer:** In incompressible flow pumps, it possible to guess the approximate rotor shape from the specific speed. Small specific speed impellers have narrow and small openings whereas large specific speed impellers have wide openings and are expected to have large flow rates. Thus, a centrifugal pump has a nearly pure radial outward flow has the small inlet area. The flow rate is small because of the small inlet area but the head against which it works is high. So for the centrifugal pumps specific speed is small. Thus, to accommodate the large flow a relatively large impeller is needed for centrifugal pumps ( $H \propto D^2$ ). A volute or mixed-flow pump has a bigger opening because of its mixed-flow characteristic though the head developed is not as large as that of the centrifugal pump. Its specific speed is higher than that of the centrifugal pump. At the extreme end is the axial-flow pump, which has a relatively large flow area and therefore a considerable volume flow rate. The head it develops is therefore small compared with that of radial-flow pumps. Its specific speed is very large.

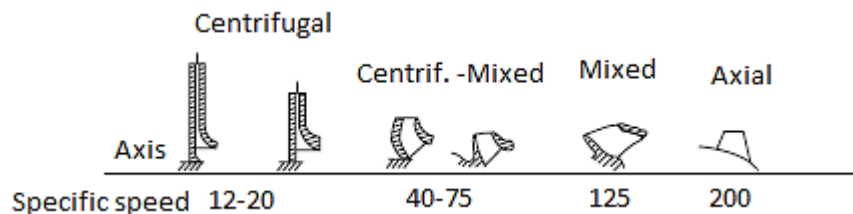


Fig. 1.2 Impeller shape variation with specific speed in pumps.

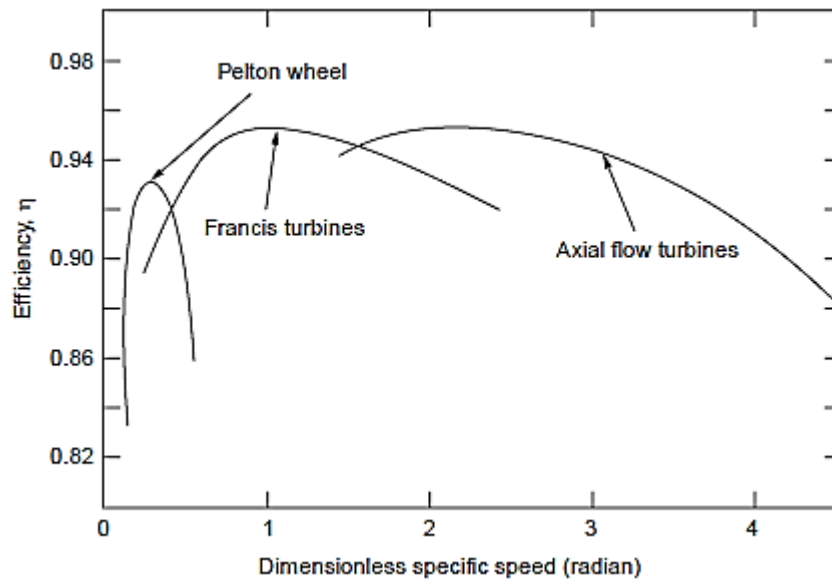


Fig. 1.3 Efficiency variation with specific speed in turbines.

Similarly, the specific speed determines the approximate shapes of the rotors as well. Consider for example the Pelton wheel which is a low specific speed, high head turbine. The volumetric flow rate is small since the turbine utilizes one or more nozzles from which the fluid emerges as jets. The Francis turbine covers a wide range of specific speeds and is suitable for intermediate heads. The Kaplan turbine operates at low heads and need large fluid flow rates to produce reasonable amounts of power. Their specific speeds are therefore high. Generally, specific speed is used as a guide to select a type of turbine under given condition of head and flow (i.e. site conditions). Therefore, such a thumb rule gives rise to a maximum efficiency. Thus, when specific speed is very high, Kaplan turbine is best selection to give rise to very high efficiency. When specific speed is very low, higher efficiencies are possible only if Pelton wheel is selected.

**1.10.4 Range of Specific Speed of Various Turbomachines:**

Specific speed in SI units		
1	Pelton wheel	
	Single jet	3 to 30
	Double jet	31 to 43
	Four jet	44 to 60
2	Francis turbine	
	Radial	61 to 102
	Mixed (Medium speed)	103 to 188
	Mixed (Fast speed)	189 to 368
3	Kaplan (Propeller) turbine	
		369 to 856
4	Centrifugal pumps	



	Turbine pump	12 to 25
	Volute pump	26 to 95
5	Mixed flow pump	96 to 210
6	Axial flow pump	211 to 320
7	Centrifugal compressor	32 to 74
8	Axial compressor	75 to 120
9	Blowers	121 to 1050

### 1.11 Unit Quantities:

**Question No 1.15: Define unit quantities. Derive expressions to each of them. (VTU, Dec-07/Jan-08)**

**Answer:** In hydraulic turbines, it is usual to define quantities as unit flow, unit speed and unit power, which are the values of the quantities under consideration per unit head.

**Unit flow ( $Q_u$ ):** Unit flow is the flow that occurs through the turbine while working under unit head.

Flow of fluid is given by,

$$Q = AC_v\sqrt{2gH} \quad (1.17)$$

Where A is area of nozzle and  $C_v$  is coefficient of velocity.

or

$$Q = K\sqrt{H} \quad (1.18)$$

Where  $K = AC_v\sqrt{2g}$  proportionality constant.

But, from definition,

$$H = 1m, Q = Q_u$$

Substitute in equation (1.18),

$$Q_u = K$$

Then, equation (1.18) can be written as,

$$Q = Q_u\sqrt{H}$$

or

$$Q_u = \frac{Q}{\sqrt{H}}$$

**Unit speed ( $N_u$ ):** Unit speed is the speed at which the machine runs under unit head.

Head coefficient is given by

$$\pi_2 = \frac{gH}{N^2D^2}$$

or

$$N^2 = KH \quad (1.19)$$

Where  $K = \frac{gH}{D^2\pi_2}$  proportionality constant.

From definition,

$$N = N_u, H = 1m$$

Substitute in equation (1.19),

$$N_u^2 = K$$

Then, equation (1.19) can be written as,

$$N^2 = N_u^2H$$

or

$$N_u = \frac{N}{\sqrt{H}}$$

**Unit power ( $P_u$ ):** Unit power is the power developed by the hydraulic machine while working under a unit head.

Power developed by hydraulic machine is given by  $P = \rho g Q H$

But, from equation (1.18),

$$Q = K\sqrt{H}$$

Then,

$$P = K\rho g H^{3/2}$$

or

$$P = C H^{3/2} \quad (1.20)$$

Where  $C = K\rho g$  proportionality constant.

From definition,

$$P = P_u, H = 1m$$

Substitute in equation (1.20),

$$P_u = C$$

Then, equation (1.20) can be written as,

$$P = P_u H^{3/2}$$

or

$$P_u = \frac{P}{H^{3/2}}$$

## 1.12 Model Studies:

The principal of all model designs is to prepare a model, from its behaviour can produce a trustworthy, consistent and accurate prediction of the prototype performance. For this prediction the model and prototype should be geometrically, kinematically and dynamically similar. Model is a small scale replica of the actual machine and the actual machine is called prototype.

**1.12.1 Geometric Similarity:** It is the similarity of form or shape. Two systems, the model and prototype are said to be geometrically similar if the ratios of all corresponding linear dimensions of the systems are equal or homologous at all points.

For geometric similarity:  $\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{d_m}{d_p}$

Where  $l$ ,  $b$  and  $d$  are the length, width and depth respectively and  $m$  and  $p$  are the suffixes that indicate model and prototype.

**1.12.2 Kinematic Similarity:** It is the similarity of motion. Two systems are considered to be kinematically similar if they are geometrically similar and ratios of components of velocity at all homologous points are equal.

For kinematic similarity:  $\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \frac{(V_3)_m}{(V_3)_p} = \dots$

Where  $(V_1)_m, (V_2)_m, (V_3)_m$  are resultant velocities at points 1, 2, and 3 in the model and  $(V_1)_p, (V_2)_p, (V_3)_p$  are resultant velocities at the corresponding points in the prototype.

**1.12.3 Dynamic Similarity:** Two systems are considered to be dynamically similar if they are geometrically and kinematically similar and the ratios of the corresponding forces acting at the corresponding points are equal.

For dynamic similarity:  $\frac{(F_1)_m}{(F_1)_p} = \frac{(F_2)_m}{(F_2)_p} = \frac{(F_3)_m}{(F_3)_p} = \dots$

Where  $(F_1)_m, (F_2)_m, (F_3)_m$  are forces acting at points 1, 2, and 3 in the model and  $(F_1)_p, (F_2)_p, (F_3)_p$  are forces acting at the corresponding points in the prototype.

### 1.13 Moody's Formula:

Machines of different sizes handling oils and other viscous fluids undergo efficiency changes under varying load conditions. For this reason, Moody has suggested an equation to determine turbine efficiencies from experiments on a geometrically similar model.

For heads smaller than 150 m, the efficiencies of model and prototype are related by the equation:

$$\eta_p = 1 - (1 - \eta_m) \left( \frac{D_m}{D_p} \right)^{0.2}$$

For heads larger than 150 m, the efficiencies of model and prototype are related by the equation:

$$\eta_p = 1 - (1 - \eta_m) \left( \frac{D_m}{D_p} \right)^{0.25} \left( \frac{H_m}{H_p} \right)^{0.1}$$

Since the power outputs for the prototype and model hydraulic turbines are  $P_p = \eta_p \rho Q_p g H_p$  and  $P_m = \eta_m \rho Q_m g H_m$ , the power-ratio may be written as:

$$\frac{P_p}{P_m} = \left( \frac{\eta_p}{\eta_m} \right) \left( \frac{Q_p}{Q_m} \right) \left( \frac{H_p}{H_m} \right)$$

It has been assumed here that similarity equations may be applied and the power incremented in proportion to the machine efficiency.

From the flow coefficient,

$$\frac{Q_p}{Q_m} = \left( \frac{N_p}{N_m} \right) \left( \frac{D_p}{D_m} \right)^3$$

But, from the head coefficient,

$$\frac{N_p}{N_m} = \left( \frac{D_m}{D_p} \right) \left( \frac{H_p}{H_m} \right)^{\frac{1}{2}}$$

Then flow-ratio may be written as,

$$\frac{Q_p}{Q_m} = \left( \frac{H_p}{H_m} \right)^{\frac{1}{2}} \left( \frac{D_p}{D_m} \right)^2$$

Finally the power-ratio may be written as,

$$\frac{P_p}{P_m} = \left( \frac{\eta_p}{\eta_m} \right) \left( \frac{D_p}{D_m} \right)^2 \left( \frac{H_p}{H_m} \right)^{\frac{3}{2}}$$

From the above relation the power output-ratio can be calculated using geometric ratio, head-ratio and efficiency-ratio.

### 1.14 Important Dimensionless Numbers:

**Question No 1.16:** Explain the following dimensionless numbers: (i) Froude's number, (ii) Weber's number, (iii) Mach's number and (iv) Euler's number. (VTU, Dec-07/Jan-08)

**Answer:**

**(i) Froude's number:** It is defined as the ratio of inertia force to gravity force. Froude's number has considerable practical significance in free surface flow problems, like flow in orifices, flow over notches, flow over the spillways etc. The flow in these problems has predominant gravitational forces.

The Froude's number is given by  $\frac{V^2}{gL}$ .

**(ii) Weber's number:** It is defined as the ratio of inertia force to the surface tension force. Weber's number has considerable practical significance in problems influenced by surface tension, like gas-liquid and liquid-liquid interfaces and contact of such interfaces with a solid boundary. These problems have predominant surface tension force.

The Weber's number is given by  $\frac{\rho LV^2}{\sigma}$ .

**(iii) Mach's number:** It is defined as the ratio of inertia force to elastic force. Mach's number has considerable practical significance in compressible flow problems, like shells, bullets, missiles and rockets fired into air. These problems have predominant elastic force.

The Mach's number is given by  $\frac{V}{\sqrt{K/\rho}}$

**(iv) Euler's number:** It is defined as the ratio of pressure force to inertia force. Euler's number has considerable practical significance in modelling of hydraulic turbines and pumps. The flow in these machines has predominant pressure forces.

The Euler's number is given by  $\frac{P}{\rho V^2}$ .

## Chapter 2

### THERMODYNAMICS OF FLUID FLOW

This chapter deals with the some basic definitions of thermodynamics applies to the turbomachines and the discussions on the thermodynamics of the fluid flow through turbomachines.

#### 2.1 Sonic Velocity and Mach Number:

**Question No 2.1:** Define Mach number and hence explain subsonic flow, sonic flow and supersonic flow. Or, write a note on Mach number. (VTU, Dec-09/Jan-10) Or,

Give classification of fluid flow based on Mach number and explain in brief. (VTU, Dec-12)

**Answer:** Sonic velocity (velocity of the sound) is referred to the speed of propagation of pressure wave in the medium. The velocity of the sound in a fluid at a local temperature  $T$  for an isentropic flow is given by

$$c = \sqrt{\gamma RT}$$

Where  $\gamma$ ,  $R$  and  $T$  are the ratio of specific heats, characteristic gas constant and the local temperature of the fluid respectively. At sea level the velocity of sound in air is given as 340 m/s.

Mach number is defined as the ratio of local velocity of fluid ( $V$ ) to the sonic velocity ( $c$ ) in that fluid. Thus

$$M = \frac{V}{c} = \frac{V}{\sqrt{\gamma RT}}$$

The fluid flow can be generally classified into subsonic flow, sonic flow and supersonic flow based on the value of Mach number.

**Subsonic flow:** If the Mach number is less than 1, then that type of flow is called subsonic flow, in which the velocity of the fluid is less than the velocity of the sound in that medium.

**Sonic flow:** If the Mach number is equal to 1, then that type of flow is called sonic flow, in which the velocity of the fluid is same as the velocity of the sound in that medium.

**Supersonic flow:** If the Mach number is greater than 1, then that type of flow is called supersonic flow, in which the velocity of the fluid is greater than the velocity of the sound in that medium.

#### 2.2 Isentropic Flow for a Varying Flow Area:

**Question No 2.2:** For the isentropic flow through varying flow area, show that  $\frac{dA}{A} = \frac{dp}{p} \left( \frac{1-M^2}{\gamma M^2} \right)$  and discuss the physical significance. Or, derive an expression for area ratio for isentropic flow through a passage of varying cross sectional area and discuss the significance of the expression. (VTU, Jun/Jul-13)

**Answer:** The Continuity equation is given by,

$$\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$$

Or,

$$\frac{dA}{A} = -\left(\frac{dV}{V} + \frac{d\rho}{\rho}\right)$$

But isentropic equation is,

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho} \Rightarrow \frac{dp}{\gamma p} = \frac{d\rho}{\rho}$$

But Euler's equation is,

$$\frac{dp}{\rho} + VdV = 0 \Rightarrow \frac{dp}{\rho V^2} + \frac{dV}{V}$$

$$\frac{dV}{V} = -\frac{dp}{\rho V^2}$$

From Mach number,

$$V^2 = M^2 \gamma RT = M^2 \gamma \left(\frac{p}{\rho}\right)$$

$$\rho V^2 = M^2 \gamma p$$

Then,

$$\frac{dV}{V} = -\frac{dp}{M^2 \gamma p} \quad (2.1)$$

Therefore,

$$\frac{dA}{A} = -\left(-\frac{dp}{M^2 \gamma p} + \frac{dp}{\gamma p}\right)$$

$$\frac{dA}{A} = \frac{dp}{M^2 \gamma p} - \frac{dp}{\gamma p} = \frac{dp}{p} \left(\frac{1}{M^2 \gamma} - \frac{1}{\gamma}\right)$$

$$\frac{dA}{A} = \frac{dp}{p} \left(\frac{1 - M^2}{\gamma M^2}\right) \quad (2.2)$$

The significance of the equations (2.1) and (2.2) is discussed below:

The equation (2.1) shows that for nozzle pressure decreases as velocity increases and for diffuser velocity decreases as pressure increases.

**For subsonic flow ( $M < 1$ )** the quantity  $\left(\frac{1-M^2}{\gamma M^2}\right)$  is positive. In the nozzle pressure decreases, so the quantity  $\frac{dp}{p}$  is negative; therefore from equation (2.2) the quantity  $\frac{dA}{A}$  is also negative and hence area must decrease for subsonic nozzle in the direction of fluid flow. The shape of the subsonic nozzle (convergent nozzle) is as shown in figure 2.1.

In the diffuser pressure increases, so the quantity  $\frac{dp}{p}$  is positive; therefore from equation (2.2) the quantity  $\frac{dA}{A}$  is also positive and hence area must increase for subsonic diffuser in the direction of fluid flow. The shape of the subsonic diffuser (divergent diffuser) is as shown in figure 2.1.

**For supersonic flow ( $M > 1$ )** the quantity  $\left(\frac{1-M^2}{\gamma M^2}\right)$  is negative. In the nozzle pressure decreases, so the quantity  $\frac{dp}{p}$  is negative; therefore from equation (2.2) the quantity  $\frac{dA}{A}$  is positive and hence area must increase for supersonic nozzle in the direction of fluid flow. The shape of the supersonic nozzle (divergent nozzle) is as shown in figure 2.2.

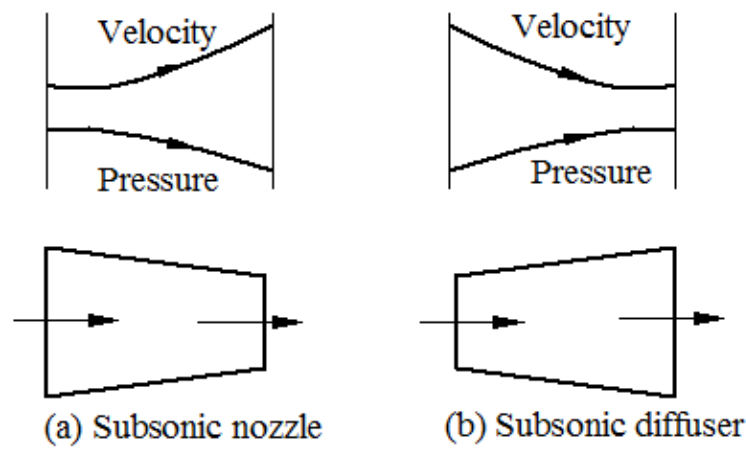


Fig. 2.1 Subsonic nozzle and diffuser

In the diffuser pressure increases, so the quantity  $\frac{dp}{p}$  is positive; therefore from equation (2.2) the quantity  $\frac{dA}{A}$  is negative and hence area must decrease for supersonic diffuser in the direction of fluid flow. The shape of the supersonic diffuser (convergent diffuser) is as shown in figure 2.2.

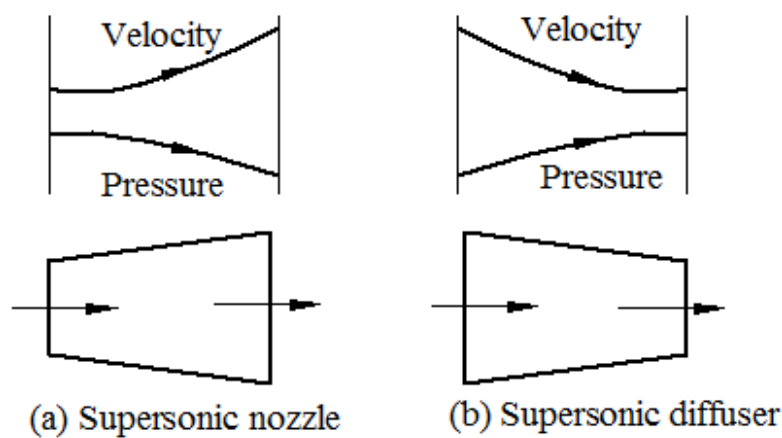


Fig. 2.2 Supersonic nozzle and diffuser

**For sonic flow (M=1)** the quantity  $\left(\frac{1-M^2}{\gamma M^2}\right)$  is zero, from equation (2.2) the quantity  $\frac{dA}{A}$  is also zero, i.e., area must be constant. This is the situation occurs at the throat portion of the convergent-divergent nozzle.

*Note: The subsonic diffuser, subsonic nozzle and the supersonic nozzle are all of practical importance as far as the turbomachines are concerned, while the supersonic diffuser is of interest for wind tunnel and ram jet.*

## 2.3 Static and Stagnation States:

**Question No 2.3: Define static state and stagnation state for a fluid.**

(VTU, Dec-11, Dec-12, Dec-14/Jan-15)

**Answer:** There are two kinds of state for the flowing fluid, namely static state and stagnation state.

**(i) Static state:** It is the state refers to those properties like pressure, temperature, density etc. which are measured when the measuring instruments are at rest relative to the flow of fluid.

**(ii) Stagnation state:** It is the final state of a fictitious, isentropic and work free process during which the final kinetic and potential energies of the fluid reduces to zero in a steady flow.

**Question No 2.4: Write expressions for (i) stagnation enthalpy, (ii) stagnation temperature, (iii) stagnation pressure and (iv) stagnation density.**

**Answer:** For a fictitious, isentropic and work free process the initial state is always the static state and final state is stagnation state. A steady flow energy equation (SFEE) for this fictitious process can be written as:

$$h_o + \frac{1}{2}V_o^2 + gZ_o + w = h + \frac{1}{2}V^2 + gZ + q$$

For isentropic and work free process,  $q=0$  and  $w=0$  and at the final state (stagnation state) of this process,  $ke=0$  and  $pe=0$ . Thus steady flow energy equation is:

$$h_o = h + \frac{1}{2}V^2 + gZ$$

**(i) Stagnation Enthalpy:** It is defined as the enthalpy of a fluid when it is adiabatically decelerated to zero velocity. The stagnation enthalpy can be written as:

$$h_o = h + \frac{1}{2}V^2 + gZ$$

Or,

$$h_o = h + \frac{1}{2}V^2$$

**(ii) Stagnation Temperature:** It is defined as the temperature of a fluid when it is adiabatically decelerated to zero velocity. The stagnation temperature defined through stagnation enthalpy as:



$$c_p T_o = c_p T + \frac{1}{2} V^2$$

$$T_o = T + \frac{V^2}{2c_p}$$

Or,

$$\frac{T_o}{T} = 1 + \frac{V^2}{2c_p T} = 1 + \frac{V^2(\gamma - 1)}{2\gamma RT} = 1 + \left(\frac{\gamma - 1}{2}\right) \frac{V^2}{c^2}$$

$$\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2$$

**(iii) Stagnation Pressure:** It is defined as the pressure of a fluid when it is adiabatically decelerated to zero velocity. The relation between the stagnation and static pressures can be written as:

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}}$$

For incompressible flows,  $h = \frac{p}{\rho}$

$$\frac{p_o}{\rho} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$p_o = p + \frac{\rho V^2}{2}$$

**(iv) Stagnation Density:** The stagnation density can be defined by using stagnation pressure and temperature. For an isentropic process,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_o}{\rho} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{1}{\gamma-1}}$$

## 2.4 Compression Process in Compressor:

### 2.4.1 Efficiency of Compression Process:

**Question No 2.5:** Define the following, with the help of a h-s diagram, for the power absorbing turbomachines: (i) Total-to-total efficiency, (ii) Total-to-static efficiency, (iii) Static-to-total efficiency, (iv) Static-to static efficiency. (VTU, Dec-06/Jan-07)

**Answer:** The h-s diagram for the compression process is shown in figure 2.3. The fluid has initially the static pressure and temperature determines by state 1, the state 01 is the corresponding stagnation state. After passing through the turbomachine, the final static properties of the fluid are determined by

state 2 and state 02 is corresponding stagnation state. If the process is reversible, the final fluid static state would be 2' while stagnation state would be 02'. Line 1-2 in static coordinates and line 01-02 in stagnation coordinates represent the real process.

The actual work input for compression process is,

$$w = h_{02} - h_{01}$$

The ideal work input can be calculated by any one of the following four equations:

(i) Total-to-total work input is the ideal work input for the stagnation ends,

$$w_{t-t} = h_{02'} - h_{01}$$

(ii) Total-to-static work input is the ideal work input for the stagnation inlet to the static exit,

$$w_{t-s} = h_{2'} - h_{01}$$

(iii) Static-to-total work input is the ideal work input for the static inlet to the stagnation exit,

$$w_{s-t} = h_{02'} - h_1$$

(iv) Static-to-static work input is the ideal work input for the static inlet to the static exit,

$$w_{s-s} = h_{2'} - h_1$$

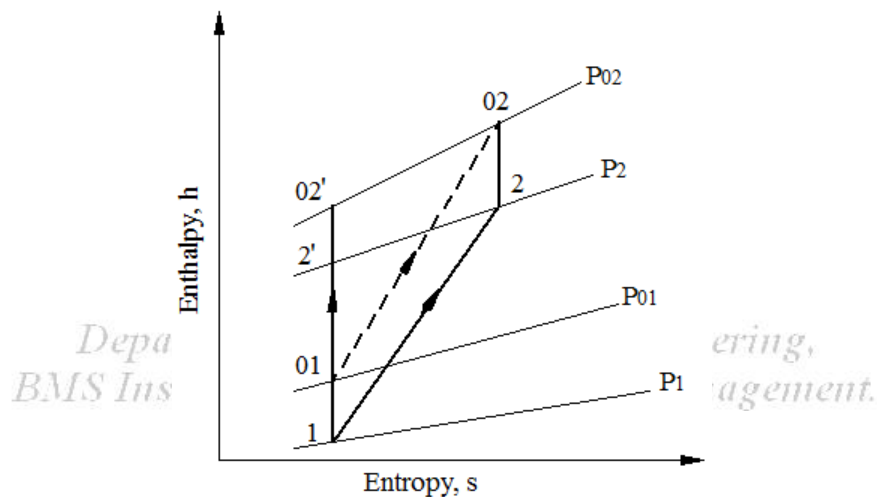


Fig. 2.3 h-s diagram for compression process

The efficiency of the compression process can be expressed by any one of the following equations:

(i) **Total-to-total efficiency** is defined as the ratio of total-to-total work input to the actual work input.

$$\eta_{t-t} = \frac{w_{t-t}}{w} = \frac{h_{02'} - h_{01}}{h_{02} - h_{01}}$$

(ii) **Total-to-static efficiency** is defined as the ratio of total-to-static work input to the actual work input.

$$\eta_{t-s} = \frac{w_{t-s}}{w} = \frac{h_{2'} - h_{01}}{h_{02} - h_{01}}$$

(iii) **Static-to-total efficiency** is defined as the ratio of static-to-total work input to the actual work input.

$$\eta_{s-t} = \frac{w_{s-t}}{w} = \frac{h_{02'} - h_1}{h_{02} - h_{01}}$$

(iv) **Static-to-static efficiency** is defined as the ratio of static-to-static work input to the actual work input.

$$\eta_{s-s} = \frac{w_{s-s}}{w} = \frac{h_{2'} - h_1}{h_{02} - h_{01}}$$

**2.4.2 Effect of Pre-heat:**

**Question No 2.6:** With the help of T-s diagram, show that the preheat factor in a multistage compressor is less than unity. (VTU, May/Jun-10, Jun/Jul-11)

**Answer:** The preheat factor for a compressor may be defined as the ratio of direct or Rankine isentropic work to the cumulative isentropic work.

The thermodynamic effect of multistage compression can be studied by considering three stage compressor working between inlet pressure  $p_1$  and the delivery pressure  $p_2$  as shown in the figure 2.4. The intermediate pressures are being  $p_A$  and  $p_B$ . The stage pressure ratio,  $p_r$  and the stage efficiency,  $\eta_{st}$  are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual compression process respectively.

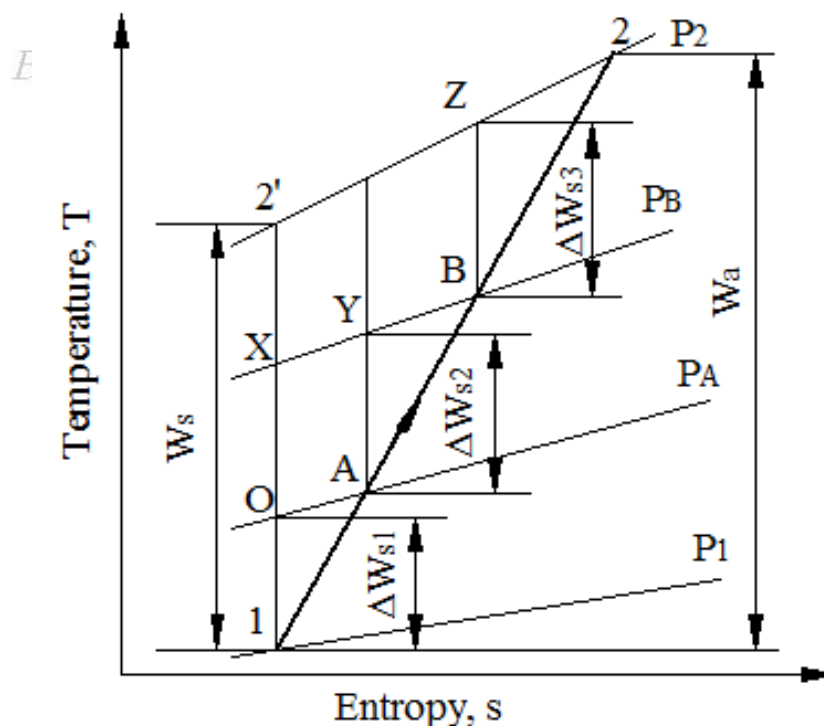


Fig. 2.4 Effect of preheat on compression process

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency. For example, in the second stage between pressures  $p_A$  and  $p_B$ , the isentropic temperature difference represented by the line A-Y is greater than that represented by the line X-O. It is therefore the isentropic work for the stage is greater by virtue of the inefficiency of the previous stage. Similarly for the next stage also.

Therefore,

$$w_s < (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

Or,

$$w_s < \Sigma \Delta w_s$$

$$\frac{w_s}{\Sigma \Delta w_s} < 1$$

Therefore, the Preheat factor  $\frac{w_s}{\Sigma \Delta w_s}$  is always less than unity for multistage compressor. This is due to the preheating of the fluid at the end of each compression stage and this appears as the losses in the subsequent stages.

**Question No 2.7:** For a multistage compressor, show that the overall efficiency is less than the stage efficiency using T-s diagram. (VTU, Jun/Jul-08)

**Answer:** Consider three stage compressor working between inlet pressure  $p_1$  and the delivery pressure  $p_2$  as shown in the figure 2.4. The intermediate pressures are being  $p_A$  and  $p_B$ . The stage pressure ratio,  $p_r$  and the stage efficiency,  $\eta_{st}$  are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual compression process respectively.

If the overall efficiency of the multistage compressor is  $\eta_o$ , then the total actual work is given by,

$$w_a = \frac{w_s}{\eta_o}$$

Or,

$$w_s = \eta_o w_a$$

The total actual work can also be written as the sum of the actual work done in each stage,

$$w_a = w_{a1} + w_{a2} + w_{a3} = \frac{\Delta w_{s1}}{\eta_{st}} + \frac{\Delta w_{s2}}{\eta_{st}} + \frac{\Delta w_{s3}}{\eta_{st}}$$

$$w_a = \frac{1}{\eta_{st}} (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

$$w_a = \frac{1}{\eta_{st}} \Sigma \Delta w_s$$

Or,

$$\Sigma \Delta w_s = \eta_{st} w_a$$

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency.

Therefore,

$$w_s < (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

Or,

$$w_s < \Sigma \Delta w_s$$

$$\eta_o w_a < \eta_{st} w_a$$

$$\eta_o < \eta_{st}$$

For multistage compressor, the overall isentropic efficiency is less than the stage efficiency.

### 2.4.3 Infinitesimal Stage Efficiency or Polytropic Efficiency:

**Question No 2.8:** Obtain an expression for polytropic efficiency for a compressor in terms of pressure ratio and temperature ratio. Further express stage efficiency in terms of polytropic efficiency and pressure ratio. Also draw the relevant T-s diagram. (VTU, Jun/Jul-13) Or, Define the term infinitesimal stage efficiency of a compressor. Show that the polytropic efficiency during the

compression process is given by  $\eta_p = \frac{\frac{\gamma-1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)}{\ln\left(\frac{T_2}{T_1}\right)}$ . (VTU, Dec-14/Jan-15)

**Answer:** A finite compressor stage is made up of number of infinitesimal stages; the efficiency of these small stages is called polytropic efficiency or infinitesimal stage efficiency.

Consider a single stage compressor having its stage efficiency  $\eta_{st}$ , operates between the pressures  $p_1$  and  $p_2$ , and an infinitesimal stage of efficiency  $\eta_p$ , working between the pressures  $p$  and  $p+dp$  as shown in figure 2.5.

The infinitesimal stage efficiency is given by,

$$\eta_p = \frac{\text{Isentropic Temperature Rise}}{\text{Actual Temperature Rise}} = \frac{dT'}{dT}$$

The actual temperature rise for infinitesimal stage is given by,

$$dT = \frac{dT'}{\eta_p} = \frac{T' - T}{\eta_p} = \frac{T \left( \frac{T'}{T} - 1 \right)}{\eta_p}$$

$$\frac{dT}{T} = \frac{\left[ \left( \frac{p+dp}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\eta_p}$$

$$\frac{dT}{T} = \frac{1}{\eta_p} \left[ \left( 1 + \frac{dp}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

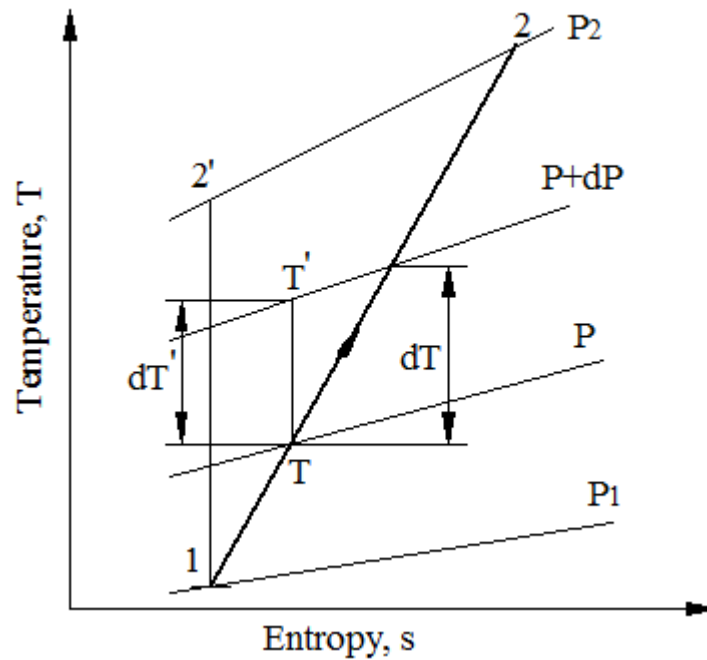


Fig. 2.5 Infinitesimal stage of a compressor

By series of expansion,  $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$  and neglecting second order differentials,

$$\frac{dT}{T} = \frac{1}{\eta_p} \left[ 1 + \frac{\gamma - 1}{\gamma} \frac{dp}{p} - 1 \right]$$

$$\frac{dT}{T} = \frac{1}{\eta_p} \frac{\gamma - 1}{\gamma} \frac{dp}{p} \tag{2.3}$$

By integration with limits 1 to 2,

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{1}{\eta_p} \frac{\gamma - 1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)$$

$$\eta_p = \frac{\frac{\gamma - 1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)}{\ln\left(\frac{T_2}{T_1}\right)}$$

**Question No 2.9:** With the help of T-s diagram, show that polytropic efficiency during the compression process is given by  $\eta_p = \left(\frac{\gamma-1}{\gamma}\right) \left(\frac{n}{n-1}\right)$  (VTU, Jun/Jul-13)

**Answer:** From equation (2.3),

$$\frac{dT}{T} = \frac{1}{\eta_p} \frac{\gamma - 1}{\gamma} \frac{dp}{p}$$

By integration,

$$\ln(T) = \frac{1}{\eta_p} \frac{\gamma - 1}{\gamma} \ln(p) + Const$$

$$\frac{p^{\frac{\gamma-1}{\eta_p \gamma}}}{T} = \text{Const}$$

For actual compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}}$$

Assume actual compression process having polytropic index 'n',

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Therefore,

$$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Equating the indices,

$$\frac{\gamma-1}{\eta_p \gamma} = \frac{n-1}{n}$$

Or,

$$\eta_p = \left(\frac{\gamma-1}{\gamma}\right) \left(\frac{n}{n-1}\right)$$

**Question No 2.10:** Derive an expression for stage efficiency of a compressor in terms of stage pressure ratio, polytropic efficiency and ratio of specific heats. Indicate the process on T-s diagram.

(VTU, Dec-12) Or,

With the help of T-s diagram, show that stage efficiency of compressor is given by

$$\eta_{st} = \frac{p_r^{\frac{\gamma-1}{\gamma}} - 1}{p_r^{\frac{\gamma-1}{\eta_p \gamma}} - 1}$$

**Answer:** From the T-s diagram shown in figure 2.5, the compressor stage efficiency is given by,

$$\eta_{st} = \frac{\text{Isentropic Temperature Rise}}{\text{Actual Temperature Rise}}$$

$$\eta_{st} = \frac{T_{2i} - T_1}{T_2 - T_1} = \frac{T_1 \left(\frac{T_{2i}}{T_1} - 1\right)}{T_1 \left(\frac{T_2}{T_1} - 1\right)} = \frac{\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}} - 1\right]}$$

$$\text{Let, } p_r = \frac{p_2}{p_1}$$

$$\eta_{st} = \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{P_r^{\eta_p \frac{\gamma-1}{\gamma}} - 1}$$

**2.4.4 Multistage Compressors:**

**Question No 2.11:** Derive an expression for an overall isentropic efficiency for multistage compression in terms of pressure ratio, polytropic efficiency, number of stages and ratio of specific heats for a compressor. Or,

Show that for a multistage compression the overall isentropic efficiency is given by

$$\eta_o = \frac{P_r^{K \frac{\gamma-1}{\gamma}} - 1}{P_r^{K \eta_p \frac{\gamma-1}{\gamma}} - 1}$$

Where  $K =$  number of stages,  $P_r =$  pressure ratio per stage,  $\eta_p =$  polytropic efficiency,  $\gamma =$  ratio of specific heats.

**Answer:** The figure 2.6 shows the T-s diagram for compression process in multistage compressor operating between the pressures  $p_1$  and  $p_{K+1}$ . If there are  $K$  stages with the overall pressure ratio  $\frac{p_{K+1}}{p_1}$  and having equal stage efficiency and stage pressure ratio.

The overall efficiency of the multistage compressor is,

$$\eta_o = \frac{\text{Total Isentropic Temperature Rise}}{\text{Total Actual Temperature Rise}}$$

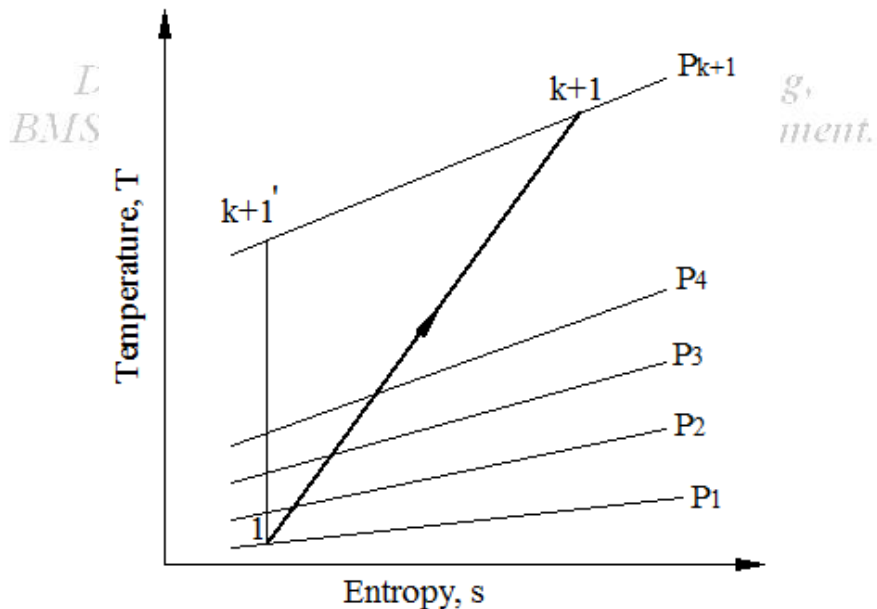


Fig. 2.6 Compression process in multistage compressor



$$\eta_o = \frac{T_{(K+1)'} - T_1}{T_{K+1} - T_1} = \frac{T_1 \left( \frac{T_{(K+1)'}}{T_1} - 1 \right)}{T_1 \left( \frac{T_{K+1}}{T_1} - 1 \right)}$$

$$\eta_o = \frac{P_{ro}^{\frac{\gamma-1}{\gamma}} - 1}{P_{ro}^{\eta_p \gamma} - 1}$$

The overall pressure ratio can be written as,  $p_{ro} = p_r^K$   
Then overall efficiency of multistage compressor is,

$$\eta_o = \frac{P_r^{K \frac{\gamma-1}{\gamma}} - 1}{P_r^{K \eta_p \gamma} - 1}$$

**Question No 2.12:** Derive an expression for an overall isentropic efficiency for finite number of stages of compression in terms of pressure ratio, stage efficiency, number of stages and ratio of specific heats for a compressor. (VTU, May/Jun-10) Or, Show that for a finite number of stages for compression the overall isentropic efficiency is given by

$$\eta_o = \frac{P_r^{K \frac{\gamma-1}{\gamma}} - 1}{\left[ 1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}} \right]^K - 1}$$

Where  $K$  = number of stages,  $P_r$  = pressure ratio per stage,  $\eta_{st}$  = stage efficiency,  $\gamma$  = ratio of specific heats. (VTU, Jan/Feb-06)

**Answer:** If  $T_1$  is the initial temperature at which the fluid enters the multistage compressor,  $K$  is the number of stages having equal pressure ratio  $p_r$  in each stage, then the actual temperature rise in each stage can be given as follows:

For first stage:

$$\Delta T_1 = (T_2 - T_1) = \frac{(T_{2'} - T_1)}{\eta_{st}} = \frac{T_1 \left( \frac{T_{2'}}{T_1} - 1 \right)}{\eta_{st}} = T_1 \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}$$

$$\text{Let } A = \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}}$$

$$\Delta T_1 = AT_1$$

For second stage:

$$\Delta T_2 = (T_3 - T_2) = AT_2 = A(T_1 + AT_1)$$

$$\Delta T_2 = AT_1(1 + A)$$

For third stage:

$$\begin{aligned}\Delta T_3 &= (T_4 - T_3) = AT_3 = A[T_2 + AT_1(1 + A)] \\ \Delta T_3 &= A[(T_1 + AT_1) + AT_1(1 + A)] = A[T_1(1 + A) + AT_1(1 + A)] \\ \Delta T_3 &= A[(1 + A)(T_1 + AT_1)] = AT_1[(1 + A)(1 + A)] \\ \Delta T_3 &= AT_1(1 + A)^2\end{aligned}$$

Similarly for fourth stage:

$$\Delta T_4 = AT_1(1 + A)^3$$

And for  $K^{\text{th}}$  stage:

$$\Delta T_K = AT_1(1 + A)^{K-1}$$

Total temperature rise across the multistage compressor is:

$$\begin{aligned}\Delta T_o &= \Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \dots + \Delta T_K \\ \Delta T_o &= AT_1 + AT_1(1 + A) + AT_1(1 + A)^2 + AT_1(1 + A)^3 + \dots + AT_1(1 + A)^{K-1} \\ \Delta T_o &= AT_1[1 + (1 + A) + (1 + A)^2 + (1 + A)^3 + \dots + (1 + A)^{K-1}] \\ \Delta T_o &= AT_1 S\end{aligned}$$

Where

$$S = 1 + (1 + A) + (1 + A)^2 + (1 + A)^3 + \dots + (1 + A)^{K-1}$$

$$S = 1 + (1 + A)[1 + (1 + A) + (1 + A)^2 \dots + (1 + A)^{K-2}]$$

Or,  $S = 1 + (1 + A)[1 + (1 + A) + (1 + A)^2 \dots + (1 + A)^{K-2} + (1 + A)^{K-1} - (1 + A)^{K-1}]$

$$S = 1 + (1 + A)[S - (1 + A)^{K-1}]$$

$$S = 1 + S(1 + A) - (1 + A)^K = 1 + S + SA - (1 + A)^K$$

$$SA = (1 + A)^K - 1$$

But,

$$\begin{aligned}\Delta T_o &= SAT_1 \\ \Delta T_o &= T_1[(1 + A)^K - 1]\end{aligned}$$

$$\Delta T_o = T_1 \left[ \left( 1 + \frac{P_r^{\frac{\gamma-1}{\eta_{st}}}}{\eta_{st}} - 1 \right)^K - 1 \right]$$

The overall efficiency of the multistage compressor is,

$$\begin{aligned}\eta_o &= \frac{\text{Total Isentropic Temperature Rise}}{\text{Total Actual Temperature Rise}} = \frac{\Delta T_o'}{\Delta T_o} \\ \eta_o &= \frac{T_{(K+1)'} - T_1}{\Delta T_o} = \frac{T_1 \left( \frac{T_{(K+1)'}}{T_1} - 1 \right)}{\Delta T_o} = \frac{T_1 \left( P_{ro}^{\frac{\gamma-1}{\eta_{st}}} - 1 \right)}{\Delta T_o}\end{aligned}$$

$$\eta_o = \frac{T_1 \left( P_r^{K \frac{\gamma-1}{\gamma}} - 1 \right)}{T_1 \left[ \left( 1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}} \right)^K - 1 \right]}$$

$$\eta_o = \frac{P_r^{K \frac{\gamma-1}{\gamma}} - 1}{\left[ 1 + \frac{P_r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{st}} \right]^K - 1}$$

## 2.5 Expansion Process in Turbine:

### 2.5.1 Efficiency of Expansion Process:

**Question No 2.13:** Define the following, with the help of a h-s diagram, for the power generating turbomachines: (i) Total-to-total efficiency, (ii) Total-to-static efficiency, (iii) Static-to-total efficiency, (iv) Static-to static efficiency. (VTU, Dec-07/Jan-08, May/Jun-10, Dec-10, Jun/Jul-11)

**Answer:** The h-s diagram for the expansion process is shown in figure 2.7. The fluid has initially the static pressure and temperature determined by state 1, the state 01 is the corresponding stagnation state. After passing through the turbomachine, the final static properties of the fluid are determined by state 2 and state 02 is corresponding stagnation state. If the process is reversible, the final fluid static state would be 2' while stagnation state would be 02'. Line 1-2 in static coordinates and line 01-02 in stagnation coordinates represent the real process.

The actual work output for expansion process is,

$$w = h_{01} - h_{02}$$

The ideal work output can be calculated by any one of the following four equations:

(i) Total-to-total work output is the ideal work output for the stagnation ends,

$$w_{t-t} = h_{01} - h_{02}$$

(ii) Total-to-static work output is the ideal work output for the stagnation inlet to the static exit,

$$w_{t-s} = h_{01} - h_2$$

(iii) Static-to-total work output is the ideal work output for the static inlet to the stagnation exit,

$$w_{s-t} = h_1 - h_{02}$$

(iv) Static-to-static work output is the ideal work output for the static inlet to the static exit,

$$w_{s-s} = h_1 - h_2$$

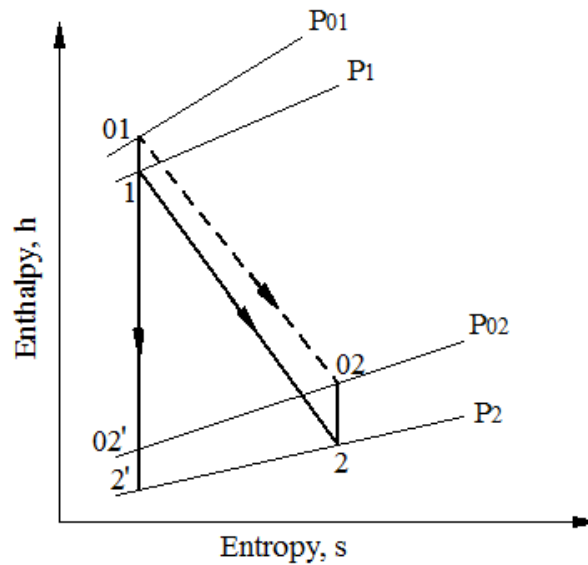


Fig. 2.7 h-s diagram for expansion process

The efficiency of the compression process can be expressed by any one of the following equations:

(i) **Total-to-total efficiency** is defined as the ratio of actual work output to the total-to-total work output.

$$\eta_{t-t} = \frac{w}{w_{t-t}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02'}}$$

(ii) **Total-to-static efficiency** is defined as the ratio of actual work output to the total-to-static work output.

$$\eta_{t-s} = \frac{w}{w_{t-s}} = \frac{h_{01} - h_{02}}{h_{01} - h_{2'}}$$

(iii) **Static-to-total efficiency** is defined as the ratio of actual work output to the static-to-total work output.

$$\eta_{s-t} = \frac{w}{w_{s-t}} = \frac{h_{01} - h_{02}}{h_1 - h_{02'}}$$

(iv) **Static-to-static efficiency** is defined as the ratio of actual work output to the static-to-static work output.

$$\eta_{s-s} = \frac{w}{w_{s-s}} = \frac{h_{01} - h_{02}}{h_1 - h_{2'}}$$

### 2.5.2 Effect of Reheat:

**Question No 2.14:** What is Reheat factor? Show that the reheat factor is greater than unity in a multistage turbine. (VTU, Dec-06/Jan-07, Jun/Jul-09, Dec-11, Jun/Jul-14, Dec-14/Jan-15)

**Answer:** The reheat factor for a turbine may be defined as the ratio of cumulative isentropic work to the direct or Rankine isentropic work.

The thermodynamic effect of multistage expansion can be studied by considering three stage turbine working between inlet pressure  $p_1$  and the delivery pressure  $p_2$  as shown in the figure 2.8. The intermediate pressures are being  $p_A$  and  $p_B$ . The stage pressure ratio,  $p_r$  and the stage efficiency,  $\eta_{st}$  are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual expansion process respectively.

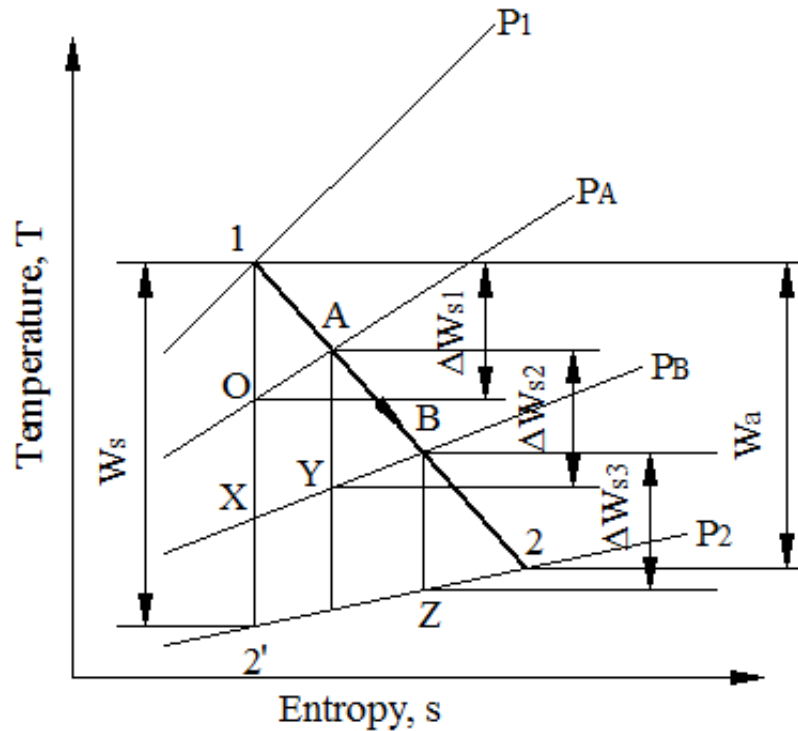


Fig. 2.8 Effect of reheat on expansion process

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency. For example, in the second stage between pressures  $p_A$  and  $p_B$ , the isentropic temperature difference represented by the line A-Y is greater than that represented by the line O-X. It is therefore the isentropic work for the stage is greater by virtue of the inefficiency of the previous stage. Similarly for the next stage also.

Therefore,

$$(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3}) > w_s$$

Or,

$$\Sigma \Delta w_s > w_s$$

$$\frac{\Sigma \Delta w_s}{w_s} > 1$$

Therefore, the Reheat factor  $\frac{\Sigma \Delta w_s}{w_s}$  is always greater than unity for multistage turbine. This is due to the reheating of the fluid at the end of each expansion stage and this appears as the losses in the subsequent stages.

**Question No 2.15:** For a multistage turbine, show that the overall efficiency is greater than the stage efficiency using T-s diagram. (VTU, Jun/Jul-08)

**Answer:** Consider three stage turbine working between inlet pressure  $p_1$  and the delivery pressure  $p_2$  as shown in the figure 2.8. The intermediate pressures are being  $p_A$  and  $p_B$ . The stage pressure ratio,  $p_r$  and the stage efficiency,  $\eta_{st}$  are assumed to be same for all stages. The process 1-2' and 1-2 are the isentropic and actual expansion process respectively.

If the overall efficiency of the multistage expansion is  $\eta_o$ , then the total actual work is given by,

$$w_a = \eta_o w_s$$

Or,

$$w_s = \frac{w_a}{\eta_o}$$

The total actual work can also be written as the sum of the actual work done in each stage,

$$w_a = w_{a1} + w_{a2} + w_{a3} = \eta_{st} \Delta w_{s1} + \eta_{st} \Delta w_{s2} + \eta_{st} \Delta w_{s3}$$

$$w_a = \eta_{st} (\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3})$$

$$w_a = \eta_{st} \Sigma \Delta w_s$$

Or,

$$\Sigma \Delta w_s = \frac{w_a}{\eta_{st}}$$

As the constant pressure lines are diverging towards the right hand side of the temperature-entropy diagram, the isentropic work per stage increases as the temperature difference increases for the same pressure ratio and the stage efficiency.

Therefore,

$$(\Delta w_{s1} + \Delta w_{s2} + \Delta w_{s3}) > w_s$$

Or,

$$\Sigma \Delta w_s > w_s$$

$$\frac{w_a}{\eta_{st}} > \frac{w_a}{\eta_o}$$

$$\eta_o > \eta_{st}$$

For multistage turbine, the overall isentropic efficiency is greater than the stage efficiency.

### 2.5.3 Infinitesimal Stage Efficiency or Polytropic Efficiency:

**Question No 2.16:** Define the term infinitesimal stage efficiency of a turbine. Show that the

polytropic efficiency during the expansion process is given by  $\eta_p = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\frac{\gamma-1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)}$

(VTU, Dec-09/Jan-10, Jun-12, Dec-13/Jan-14)

**Answer:** A finite turbine stage is made up of number of infinitesimal stages; the efficiency of these small stages is called polytropic efficiency or infinitesimal stage efficiency.

Consider a single stage turbine having its stage efficiency  $\eta_{st}$ , operates between the pressures  $p_1$  and  $p_2$ , and an infinitesimal stage of efficiency  $\eta_p$ , working between the pressures  $p$  and  $p-dp$  as shown in figure 2.9.

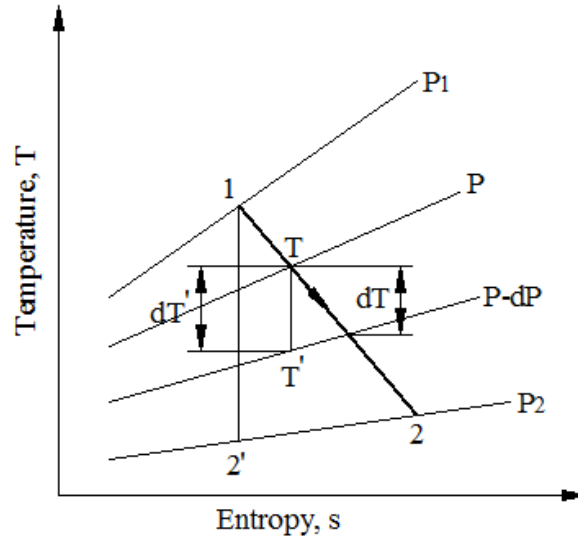


Fig. 2.9 Infinitesimal stage of a turbine

The infinitesimal stage efficiency is given by,

$$\eta_p = \frac{\text{Actual Temperature Drop}}{\text{Isentropic Temperature Drop}} = \frac{dT}{dT'}$$

The actual temperature rise for infinitesimal stage is given by,

$$dT = \eta_p dT' = \eta_p (T - T') = \eta_p T \left( 1 - \frac{T'}{T} \right)$$

$$\frac{dT}{T} = \eta_p \left[ 1 - \left( \frac{p-dp}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{dT}{T} = \eta_p \left[ 1 - \left( 1 - \frac{dp}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

By series of expansion,  $(1 - x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \dots$  and neglecting second order differentials,

$$\frac{dT}{T} = \eta_p \left[ 1 - \left( 1 - \frac{\gamma-1}{\gamma} \frac{dp}{p} \right) \right]$$

$$\frac{dT}{T} = \eta_p \frac{\gamma-1}{\gamma} \frac{dp}{p} \tag{2.4}$$

By integration with limits 1 to 2,

$$\ln\left(\frac{T_2}{T_1}\right) = \eta_p \frac{\gamma - 1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)$$

$$\eta_p = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\frac{\gamma - 1}{\gamma} \ln\left(\frac{p_2}{p_1}\right)}$$

Or,

$$\eta_p = \frac{\frac{\gamma}{\gamma - 1} \ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{p_2}{p_1}\right)}$$

**Question No 2.17:** With the help of T-s diagram, show that polytropic efficiency during expansion process is given by  $\eta_p = \left(\frac{\gamma}{\gamma - 1}\right) \left(\frac{n - 1}{n}\right)$  (VTU, Dec-08/Jan-09, Jun/Jul-14)

**Answer:** From equation (2.4),

$$\frac{dT}{T} = \eta_p \frac{\gamma - 1}{\gamma} \frac{dp}{p}$$

By integration,

$$\ln(T) = \eta_p \frac{\gamma - 1}{\gamma} \ln(p) + \text{Const}$$

$$\frac{p^{\eta_p \frac{\gamma - 1}{\gamma}}}{T} = \text{Const}$$

For actual compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_p \frac{\gamma - 1}{\gamma}}$$

Assume actual compression process having polytropic index 'n',

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$$

Therefore,

$$\left(\frac{p_2}{p_1}\right)^{\eta_p \frac{\gamma - 1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$$

Equating the indices,

$$\eta_p \frac{\gamma - 1}{\gamma} = \frac{n - 1}{n}$$

Or,

$$\eta_p = \left(\frac{\gamma}{\gamma - 1}\right) \left(\frac{n - 1}{n}\right) \quad (2.5)$$



**Question No 2.18:** Show that the index 'n' of polytropic expansion in a turbine of infinitesimal stage efficiency  $\eta_p$  is given by  $n = \frac{\gamma}{\gamma - (\gamma - 1)\eta_p}$ , where  $\gamma$  is a ratio of specific heats. (VTU, Dec-10)

**Answer:** From equation (2.5),

$$\begin{aligned}\eta_p &= \left(\frac{\gamma}{\gamma - 1}\right) \left(\frac{n - 1}{n}\right) \\ \eta_p(\gamma - 1)n &= \gamma(n - 1) \\ \eta_p\gamma n - \eta_p n &= \gamma n - \gamma \\ \eta_p\gamma n - \eta_p n - \gamma n &= -\gamma \\ n(\eta_p\gamma - \eta_p - \gamma) &= -\gamma \\ n &= \frac{-\gamma}{\eta_p\gamma - \eta_p - \gamma} = \frac{\gamma}{-\eta_p\gamma + \eta_p + \gamma} \\ n &= \frac{\gamma}{\gamma - (\gamma - 1)\eta_p}\end{aligned}$$

**Question No 2.19:** With the help of T-s diagram, show that stage efficiency of turbine is given by

$$\eta_{st} = \frac{1 - P_r^{-(\eta_p \frac{\gamma-1}{\gamma})}}{1 - P_r^{-\frac{\gamma-1}{\gamma}}} \quad (\text{VTU, Jun-12})$$

**Answer:** From the T-s diagram shown in figure 2.9, the turbine stage efficiency is given by,

$$\begin{aligned}\eta_{st} &= \frac{\text{Actual Temperature Drop}}{\text{Isentropic Temperature Drop}} \\ \eta_{st} &= \frac{T_1 - T_2}{T_1 - T_{2'}} = \frac{T_1 \left(1 - \frac{T_2}{T_1}\right)}{T_1 \left(1 - \frac{T_{2'}}{T_1}\right)} = \frac{\left[1 - \left(\frac{p_2}{p_1}\right)^{\eta_p \frac{\gamma-1}{\gamma}}\right]}{\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\end{aligned}$$

$$\text{Let, } p_r = \frac{p_1}{p_2}$$

$$\eta_{st} = \frac{1 - P_r^{-(\eta_p \frac{\gamma-1}{\gamma})}}{1 - P_r^{-\frac{\gamma-1}{\gamma}}}$$

### 2.5.4 Multistage Turbines:

**Question No 2.20:** Derive an expression for an overall isentropic efficiency for multistage expansion in terms of pressure ratio, polytropic efficiency, number of stages and ratio of specific heats for a turbine. Or,

Show that for a multistage expansion the overall isentropic efficiency is given by

$$\eta_o = \frac{1 - P_r^{-(\eta_p \frac{\gamma-1}{\gamma} K)}}{1 - P_r^{-\frac{(\gamma-1)K}{\gamma}}}$$

Where  $K$ = number of stages,  $P_r$ = pressure ratio per stage,  $\eta_p$ = polytropic efficiency,  $\gamma$ = ratio of specific heats.

**Answer:** The figure 2.10 shows the T-s diagram expansion process in multistage turbine operating between the pressures  $p_1$  and  $p_{K+1}$ . If there are  $K$  stages with the overall pressure ratio  $\frac{p_1}{p_{K+1}}$  and having equal stage efficiency and stage pressure ratio.

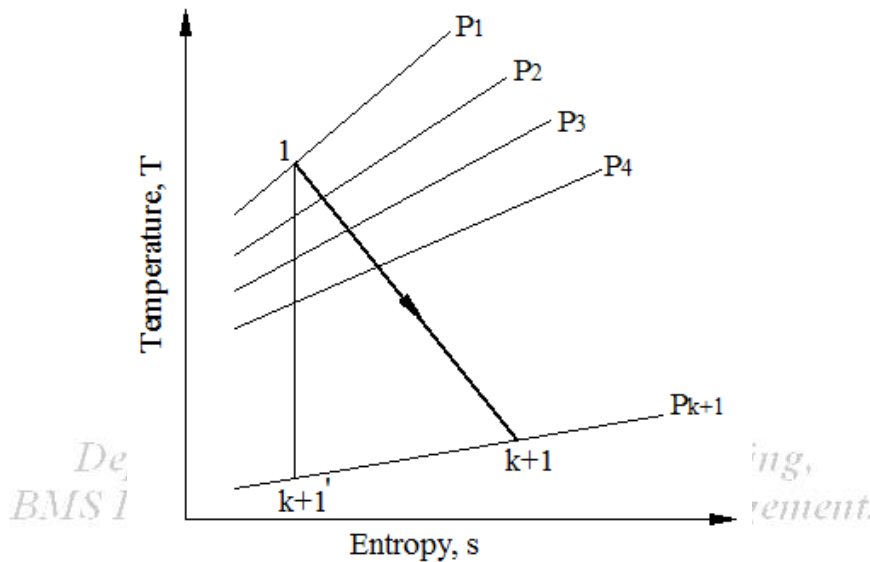


Fig. 2.10 Expansion process in multistage turbine

The overall efficiency of the multistage turbine is,

$$\eta_o = \frac{\text{Total Actual Temperature Drop}}{\text{Total Isentropic Temperature Drop}}$$

$$\eta_o = \frac{T_1 - T_{K+1}}{T_1 - T_{(K+1)'}} = \frac{T_1 \left(1 - \frac{T_{K+1}}{T_1}\right)}{T_1 \left(1 - \frac{T_{(K+1)'}}{T_1}\right)} = \frac{\left[1 - \left(\frac{p_{K+1}}{p_1}\right)^{\eta_p \frac{\gamma-1}{\gamma}}\right]}{\left[1 - \left(\frac{p_{(K+1)'}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Let,  $p_{ro} = \frac{p_1}{p_{K+1}}$

$$\eta_o = \frac{1 - P_{ro}^{-\left(\eta_p \frac{\gamma-1}{\gamma}\right)}}{1 - P_{ro}^{-\left(\frac{\gamma-1}{\gamma}\right)}}$$

The overall pressure ratio can be written as,  $p_{ro} = p_r^K$

Then overall efficiency of multistage turbine is,

$$\eta_o = \frac{1 - P_r^{-\left(\eta_p \frac{\gamma-1}{\gamma} K\right)}}{1 - P_r^{-\left(\frac{\gamma-1}{\gamma} K\right)}}$$

**Question No 2.21:** Derive an expression for an overall isentropic efficiency for finite number of stages of expansion in terms of pressure ratio, stage efficiency, number of stages and ratio of specific heats for a turbine. (VTU, Jul-07) Or,

Show that for a finite number of stages for expansion the overall isentropic efficiency is given by

$$\eta_o = \frac{1 - \left\{ 1 - \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^K}{\left[ 1 - \left( \frac{1}{p_r} \right)^{K \frac{\gamma-1}{\gamma}} \right]}$$

Where  $K =$  number of stages,  $P_r =$  pressure ratio per stage,  $\eta_{st} =$  stage efficiency,  $\gamma =$  ratio of specific heats. (VTU, Jun/Jul-09)

**Answer:** If  $T_1$  is the initial temperature at which the fluid enters the multistage turbine,  $K$  is the number of stages having equal pressure ratio  $p_r$  in each stage, then the actual temperature drop in each stage can be given as follows:

For first stage:

$$\Delta T_1 = (T_1 - T_2) = \eta_{st}(T_1 - T_{2'}) = T_1 \eta_{st} \left( 1 - \frac{T_{2'}}{T_1} \right) = T_1 \eta_{st} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\Delta T_1 = T_1 \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\text{Let, } B = \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\Delta T_1 = B T_1$$

For second stage:

$$\Delta T_2 = (T_2 - T_3) = B T_2 = B(T_1 - B T_1)$$

$$\Delta T_2 = B T_1 (1 - B)$$

For third stage:

$$\Delta T_3 = (T_3 - T_4) = B T_3 = B[T_2 - B T_1 (1 - B)]$$

$$\Delta T_3 = B[T_1 - BT_1 - BT_1(1 - B)] = BT_1[(1 - B) - B(1 - B)]$$

$$\Delta T_3 = BT_1(1 - B)^2$$

Similarly for fourth stage:

$$\Delta T_4 = BT_1(1 - B)^3$$

And for  $K^{\text{th}}$  stage:

$$\Delta T_K = BT_1(1 - B)^{K-1}$$

Total temperature drop across the multistage turbine is:

$$\Delta T_o = \Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 + \dots + \Delta T_K$$

$$\Delta T_o = BT_1 + BT_1(1 - B) + BT_1(1 - B)^2 + BT_1(1 - B)^3 + \dots + BT_1(1 - B)^{K-1}$$

$$\Delta T_o = BT_1[1 + (1 - B) + (1 - B)^2 + (1 - B)^3 + \dots + (1 - B)^{K-1}]$$

Let,

$$S = 1 + (1 - B) + (1 - B)^2 + (1 - B)^3 + \dots + (1 - B)^{K-1}$$

$$S = 1 + (1 - B)[1 + (1 - B) + (1 - B)^2 + \dots + (1 - B)^{K-2}]$$

$$S = 1 + (1 - B)[1 + (1 - B) + (1 - B)^2 + \dots + (1 - B)^{K-2} + (1 - B)^{K-1} - (1 - B)^{K-1}]$$

$$S = 1 + (1 - B)[S - (1 - B)^{K-1}] = 1 + S(1 - B) - (1 - B)^K$$

$$S = 1 + S - SB - (1 - B)^K$$

$$SB = 1 - (1 - B)^K$$

But,

$$\Delta T_o = SBT_1$$

$$\Delta T_o = T_1[1 - (1 - B)^K]$$

$$\Delta T_o = T_1 \left\{ 1 - \left[ 1 - \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]^K \right] \right\}$$

The overall efficiency of the multistage turbine is,

$$\eta_o = \frac{\text{Total Actual Temperature Drop}}{\text{Total Isentropic Temperature Drop}}$$

$$\eta_o = \frac{\Delta T_o}{T_1 - T_{(K+1)'}} = \frac{T_1 \left\{ 1 - \left[ 1 - \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]^K \right] \right\}}{T_1 \left( 1 - \frac{T_{(K+1)'}}{T_1} \right)} = \frac{1 - \left[ 1 - \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]^K \right]}{\left[ 1 - \left( \frac{p_{(K+1)'}}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

But,  $\frac{p_1}{p_{(K+1)'}} = p_{ro} = p_r^K$

$$\eta_o = \frac{1 - \left\{ 1 - \eta_{st} \left[ 1 - \left( \frac{1}{p_r} \right)^{\frac{\gamma-1}{\gamma}} \right]^K \right\}}{\left[ 1 - \left( \frac{1}{p_r} \right)^{K \frac{\gamma-1}{\gamma}} \right]}$$



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## Chapter 3

### ENERGY TRANSFER IN TURBOMACHINES

#### 3.1 Introduction:

In this chapter, general analysis of kinematic and dynamic factors for different types of turbomachines is made. Kinematics relates to movement (velocities, accelerations, etc.), without paying attention to what brought about the motion. Dynamics is related to detailed examination of the forces that bring about the motion described by kinematics. The kinematic and dynamic factors depend on the velocities of fluid flow in the machine as well as the rotor velocity itself and the forces of interaction due to velocity changes.

#### 3.2 Euler's Turbine Equation:

**Question No 3.1:** *Derive Euler's turbine equation for power generating or power absorbing turbomachines and clearly state the assumptions made. (VTU, Jan/Feb-03, Dec-12)*

**Answer:** The figure 3.1 shows the rotor of a generalized turbomachine with axis of rotation  $O-O$ , with an angular velocity  $\omega$ . The fluid enters the rotor at radius  $r_1$  with an absolute velocity  $V_1$  and leaves the rotor at radius  $r_2$  with an absolute velocity  $V_2$ .

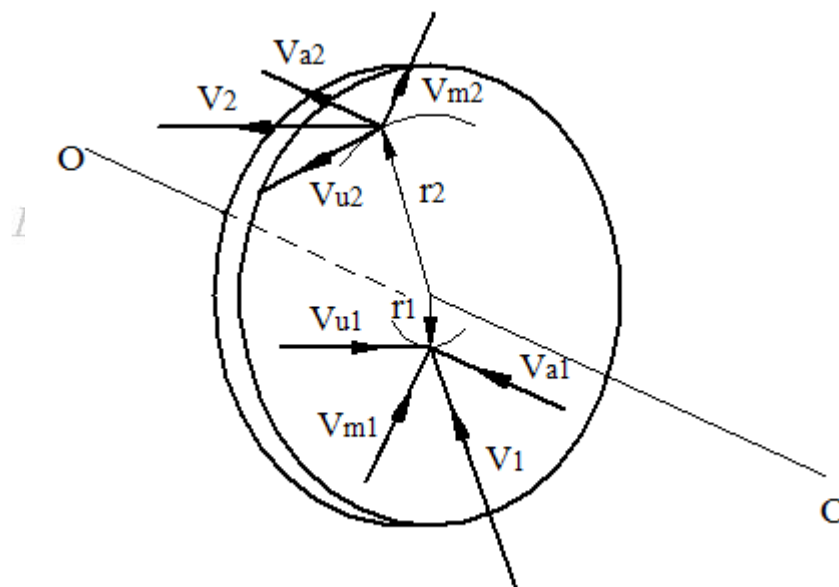


Fig. 3.1 Fluid flow through a rotor of a turbomachine.

*Assumptions:*

- i. Fluid flow through the turbomachine is steady flow.
- ii. Mass flow rate is constant and the state of the fluid doesn't vary with time.
- iii. Rate of energy transfer at the rotor is constant.

iv. Losses due to leakage are neglected.

The absolute velocity of the fluid can be resolved in to:

- Axial component ( $V_a$ ), which is parallel to the axis of rotation of the rotor.
- Radial component ( $V_m$ ), which is perpendicular to the axis of rotation of the rotor.
- Tangential component ( $V_u$ ), which is along the tangential direction of the rotor.

The only velocity component which changes the angular momentum of the rotor is the tangential component ( $V_u$ ) and by Newton's second law of motion forces applied on the rotor is equal to rate of change of momentum of the fluid.

Force applied on the rotor = Rate of change of momentum

$$F = \Delta \left( \frac{mV_u}{t} \right) = \dot{m}(V_{u1} - V_{u2})$$

But, Torque = Force  $\times$  Radius

$$\tau = F \times r$$

Then,  $\tau = \dot{m}(V_{u1}r_1 - V_{u2}r_2)$

But, Rate of energy transfer = Torque  $\times$  Angular velocity

$$\dot{E} = \tau \times \omega$$

Then,

$$\dot{E} = \dot{m}(V_{u1}r_1\omega_1 - V_{u2}r_2\omega_2)$$

But, tangential velocity of rotor

$$U = r \times \omega$$

Then,

$$\dot{E} = \dot{m}(U_1V_{u1} - U_2V_{u2})$$

Energy transfer per unit mass flow of fluid is

$$e = \frac{\dot{E}}{\dot{m}} = (U_1V_{u1} - U_2V_{u2}) \quad (3.1)$$

The equation (3.1) is the general Euler's equation for all kind of turbomachines.

For power generating turbomachine energy transfer is positive (i.e.,  $U_1V_{u1} > U_2V_{u2}$ )

Therefore,

$$e = (U_1V_{u1} - U_2V_{u2}) \quad (3.2)$$

For power absorbing turbomachine energy transfer is negative (i.e.,  $U_2V_{u2} > U_1V_{u1}$ )

Therefore,

$$e = (U_2V_{u2} - U_1V_{u1}) \quad (3.3)$$

*Note: (a) The change in magnitude of axial velocity components give rise to an axial thrust which must be taken up by the thrust bearings. The change in magnitude of radial velocity components give rise to a radial thrust which must be taken up by the journal bearing. Neither of these forces causes any angular rotation nor has any effect on the torque exerted on the rotor.*

*(b) The Euler's turbine equation may be used for the flow of fluids like water, steam, air and combustion products, since their viscosities are reasonably small. For fluids of very large viscosity like heavy oils or petroleum products, errors in the calculated torque and power output may result due to: (i) non-uniformity of velocity profiles at the inlet and the exit and (ii) the boundary layers near the housing and the stator surfaces. Both these tend reduce the magnitude of the torque in comparison with the ideal torque predicted by Euler's turbine equation.*

### 3.3.1 Procedure to Draw Velocity Diagram:

**Question No 3.2:** Explain the procedure to draw velocity triangles. Why velocity triangles are of utmost importance in the study of turbomachines? (VTU, Dec-10)

**Answer:** In turbomachinery, a velocity triangle or a velocity diagram is a triangle representing the various components of velocities of the working fluid in a turbomachine. Velocity triangles may be drawn for both the inlet and outlet sections of any turbomachine. The vector nature of velocity is utilized in the triangles, and the most basic form of a velocity triangle consists of the tangential velocity, the absolute velocity and the relative velocity of the fluid making up three sides of the triangle.

Consider turbomachine consisting of a stator and a rotor. The three points that are very much important to draw the velocity triangles are entry to the stator, the gap between the stator and rotor and exit from the rotor. These points labelled 3, 1 and 2 respectively in figure 3.2 and combination of rotor and stator is called stage in turbomachines.

The fluid enters the stator at point 3 but as the stator is not moving there is no relative motion between the incoming flow and the stator so there is no velocity triangle to draw at this point. At point 1 the flow leaves the stator and enters the rotor. Here there are two flow velocities, the absolute velocity of the flow ( $V$ ) viewed from the point of view of stationary stator and relative velocity of flow ( $V_r$ ) viewed from the point of view of moving rotor. The rotor is moving with a tangential velocity of magnitude  $U$ . At point 2 the flow leaves the rotor and exits the stage. Again there are two flow velocities, one by viewing from the moving rotor and another by viewing from outside the rotor where there is no motion.

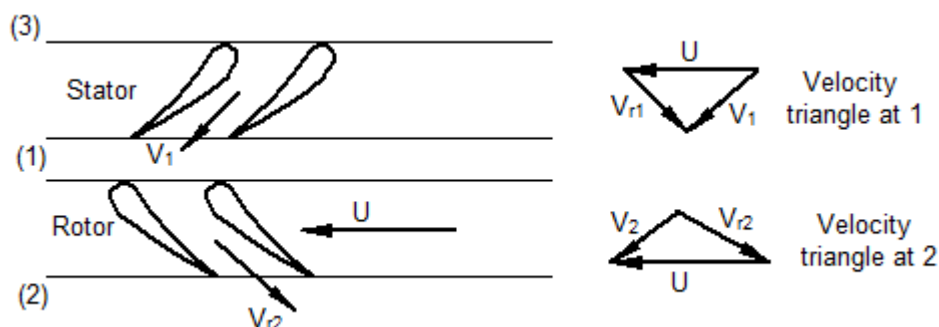


Fig.3.2 Velocity triangles for a turbomachine.

Therefore velocity triangles can be drawn for the point 1 and point 2 as shown in figure 3.2, the methodology for this is as follows:

1. Draw the flow that is known



2. Draw the blade speed
3. Close the triangle with the remaining vector
4. Check that the key rule applies:  $\vec{V} = \vec{U} + \vec{V}_r$

The velocity triangles at inlet and outlet of the rotor are utmost important in deciding the size of the turbomachine for the given power output.

### 3.2.2 Energy components of Euler's Turbine Equation:

**Question No 3.3:** Derive an alternate (modified) form of Euler's turbine equation with usual notations and identify each component contained in the equation. (VTU, Jun/Jul-09, Dec-13/Jan-14, Jun/Jul-14) Or, Draw the velocity triangle at inlet and exit of a turbomachine in general and show that the energy transfer per unit mass is given by  $e = \frac{1}{2} [(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]$  (VTU, Feb-06, Jun/Jul-13)

**Answer:** Let us consider velocity diagram for generalised rotor as shown in figure 3.3.

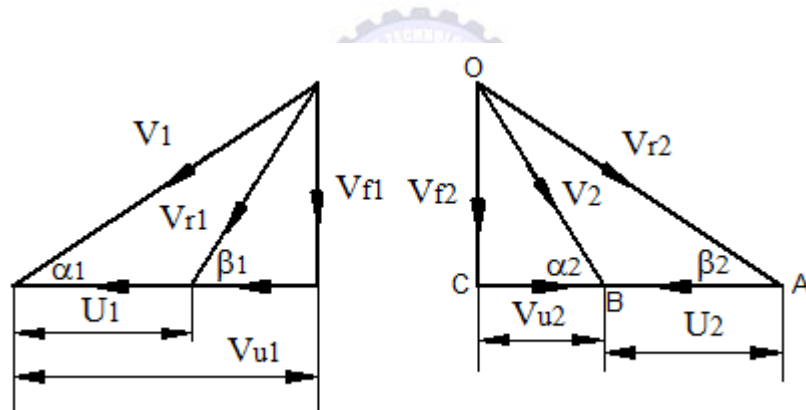


Fig. 3.3 Generalised velocity diagrams.

Let  $V$  = Absolute velocity of fluid

$\alpha$  = Angle made by  $V$  wrt tangential direction or nozzle angle or guide vane angle

$V_r$  = Relative velocity of the fluid

$\beta$  = Rotor angle or blade angle wrt tangential direction

$U$  = Tangential velocity of the rotor

$V_u$  = Tangential component of the absolute velocity or whirl velocity

$V_f = V_m = V_a$  = Radial component or axial component of the absolute velocity or flow velocity.

Suffix 1 and 2 represents the values at inlet and outlet of the rotor.

Consider outlet velocity triangle, OBC

$$V_{f2}^2 = V_2^2 - V_{u2}^2 \quad (3.4)$$

From outlet velocity triangle, OAC

$$V_{f2}^2 = V_{r2}^2 - (U_2 - V_{u2})^2 \quad (\text{Because, } U_2 \text{ and } V_{u2} \text{ are in opposite direction})$$

$$V_{f2}^2 = V_{r2}^2 - U_2^2 - V_{u2}^2 + 2U_2V_{u2} \quad (3.5)$$

Compare equations (3.4) and (3.5)

$$V_2^2 - V_{u2}^2 = V_{r2}^2 - U_2^2 - V_{u2}^2 + 2U_2V_{u2}$$

$$2U_2V_{u2} = V_2^2 + U_2^2 - V_{r2}^2$$

Or

$$U_2V_{u2} = \frac{1}{2}(V_2^2 + U_2^2 - V_{r2}^2) \quad (3.6)$$

Similarly, for inlet velocity triangle

$$U_1V_{u1} = \frac{1}{2}(V_1^2 + U_1^2 - V_{r1}^2) \quad (3.7)$$

Substitute equations (3.6) and (3.7) in Euler's turbine equations (3.2) and (3.3)

For power generating turbomachines,

$$e = \frac{1}{2}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)] \quad (3.8)$$

For power absorbing turbomachines,

$$e = \frac{1}{2}[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) - (V_{r2}^2 - V_{r1}^2)] \quad (3.9)$$

**First component:**  $\frac{(V_1^2 - V_2^2)}{2}$  or  $\frac{(V_2^2 - V_1^2)}{2}$  change in the absolute kinetic energy and which causes a change in the dynamic head or dynamic pressure of the fluid through the machine.

**Second component:**  $\frac{(U_1^2 - U_2^2)}{2}$  or  $\frac{(U_2^2 - U_1^2)}{2}$  change in the centrifugal energy of the fluid in the motion. This is due to the change in the radius of rotation of the fluid. This causes a change in the static head or static pressure of the fluid through the rotor.

**Third component:**  $\frac{(V_{r1}^2 - V_{r2}^2)}{2}$  or  $\frac{(V_{r2}^2 - V_{r1}^2)}{2}$  change in the relative kinetic energy and which causes a change in the static head or static pressure of the fluid across the rotor.

Note: If directions of  $V_{u1}$  and  $V_{u2}$  are same then,  $e = (U_1V_{u1} - U_2V_{u2})$  and if directions of  $V_{u1}$  and  $V_{u2}$  are opposite to each other then,  $e = (U_1V_{u1} + U_2V_{u2})$ .

Dynamic pressure is the kinetic energy per unit volume of a fluid particle. The dynamic pressure is equal to the difference between the stagnation pressure and static pressure. Dynamic pressure sometimes called velocity pressure. Static pressure is the actual pressure of the fluid, which is associated not with its motion but with its state. Stagnation or total pressure is sum of static pressure and dynamic pressure.

$$P_o = P + \frac{\rho V^2}{2}$$

### 3.3 General Analysis of Turbomachines:

**3.3.1 Impulse and Reaction Turbomachines:** In general, turbomachines may be classified into impulse and reaction types, depending upon the type of energy exchange that occurs in the rotor blades. An impulse stage is one in which the static pressure at the rotor inlet is the same as that at the

rotor outlet (i.e.  $V_{r1} = V_{r2}$  and  $U_1 = U_2$ ). In an impulse stage, the energy exchange is purely due to change in the direction of the fluid (i.e., change in dynamic pressure) and there is a negligible change in the magnitude of velocity as fluid flows over the rotor blades. The force exerted on the blades is due to change in the direction of the fluid during flow over the moving blade.

A reaction stage is one where a change in static pressure occurs during flow over each rotor stage. In a reaction stage, the direction and magnitude of the relative velocity are changed by shaping the blade passage as a nozzle (or as a diffuser, depending upon whether it is generating or absorbing power). The force exerted on the blades is due to both changes in magnitude and in direction of the fluid velocity.

**3.3.2 Degree of Reaction (R):** The degree of reaction is a parameter which describes the relation between the energy transfer due to static pressure change and the energy transfer due to dynamic pressure change. The *degree of reaction is the ratio of energy transfer due to the change in static pressure in the rotor to total energy transfer due to the change in total pressure in the rotor.*

Mathematically,

$$R = \frac{\frac{1}{2}[(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}{\frac{1}{2}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}$$

Or

$$R = \frac{e - \frac{1}{2}(V_1^2 - V_2^2)}{e}$$

**3.3.3 Utilization Factor ( $\epsilon$ ):**

**Question No 3.4: Define utilization factor and derive an expression for the same for a power developing turbomachines. (VTU, Jan/Feb-03)**

**Answer:** The utilization factor is the ratio of the ideal (Euler) work output to the energy available for conversion into work. Under ideal conditions, it should be possible to utilize all of the kinetic energy of the fluid at the rotor inlet and also the increase in kinetic energy obtained in the rotor due to static pressure drop (i.e. the reaction effect). Thus, the energy available for conversion into work in the turbine is:  $e_a = \frac{1}{2}[V_1^2 + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]$

On the other hand, Euler work output is:  $e = \frac{1}{2}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]$

Mathematically, utilization factor is:

$$\epsilon = \frac{\frac{1}{2}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}{\frac{1}{2}[V_1^2 + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}$$

Or

$$\epsilon = \frac{e}{e + \frac{V_2^2}{2}}$$

**Question No 3.5:** Define utilization factor for a turbine. If the isentropic efficiency of a turbine is 100% would its utilization factor also be 100%? Explain.

**Answer:** The utilization factor is the ratio of the ideal (Euler) work output to the energy available for conversion into work.

Yes, because the isentropic efficiency (adiabatic efficiency) is the product of two factors, the first called the utilization factor (diagram efficiency), the second due to non-isentropic flow conditions caused by friction, turbulence, eddies and other losses. Therefore if the isentropic efficiency has to be 100%, the utilization factor must be 100%.

**Question No 3.6:** Derive an expression relating utilization factor with the degree of reaction. Or, Show that utilization factor is given by  $\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$  where  $R$  is the degree of reaction. For what value of  $R$  this relation is invalid? Why? (VTU, Jan/Feb-03, Jul/Aug-05, Dec-08/Jan-09, Dec-12, Jun/Jul-13, Dec-14/Jan-15)

**Answer:** Degree of reaction for generalised turbomachine is given by:

$$R = \frac{[(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}{[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}$$

$$R(V_1^2 - V_2^2) + R[(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)] = (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)$$

$$R(V_1^2 - V_2^2) = (1 - R)[(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]$$

Then,

$$(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2) = \frac{R}{(1-R)}(V_1^2 - V_2^2) \quad (3.10)$$

The utilization factor for any type of turbine is given by:

$$\epsilon = \frac{[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}{[V_1^2 + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)]}$$

From equation (3.10)

$$\epsilon = \frac{(V_1^2 - V_2^2) + \frac{R}{(1-R)}(V_1^2 - V_2^2)}{V_1^2 + \frac{R}{(1-R)}(V_1^2 - V_2^2)}$$

$$\epsilon = \frac{(1-R)(V_1^2 - V_2^2) + R(V_1^2 - V_2^2)}{(1-R)V_1^2 + R(V_1^2 - V_2^2)}$$

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

The above equation is the general utilization factor irrespective of any type of turbines whether it is axial or radial type. Clearly, it is invalid when  $R=1$ , since  $\epsilon=1$ . Therefore the above equation is valid for all values of  $R$  in the range of  $0 \leq R < 1$ .

**3.3.4 Condition for Maximum Utilization Factor:**

**Question No 3.7:** In a turbomachine, prove that the maximum utilization factor is given by

$$\epsilon_{max} = \frac{2\phi \cos\alpha_1}{1+2R\phi \cos\alpha_1}, \text{ where } \phi \text{ is speed ratio, } R \text{ is degree of reaction and } \alpha_1 \text{ is nozzle angle.}$$

(VTU, Jan/Feb-05, Dec-11)

**Answer:** For maximum utilization, the value of  $V_2$  should be the minimum and from the velocity triangle, it is apparent that  $V_2$  is having minimum value when it is axial or radial (i.e.,  $V_2=V_{f2}$ ). Then the velocity diagram of generalized turbomachine for maximum utilization is as shown in figure 3.4.

Energy transfer of a generalized turbomachine is given by:

$$e = \frac{1}{2} [(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)] = (U_1 V_{u1} - U_2 V_{u2})$$

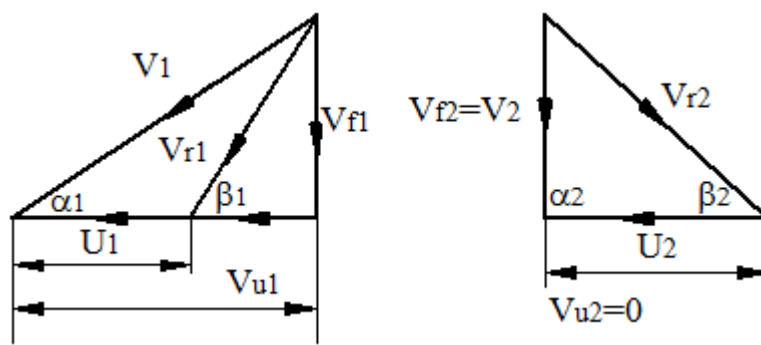


Fig. 3.4 Velocity diagram of generalized turbomachine for maximum utilization

For maximum utilization  $V_{u2}=0$ ,

$$\frac{1}{2} [(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)] = U_1 V_{u1}$$

From equation (3.10),  $(U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2) = \frac{R}{(1-R)} (V_1^2 - V_2^2)$

Then,

$$\frac{1}{2} \left[ (V_1^2 - V_2^2) + \frac{R}{(1-R)} (V_1^2 - V_2^2) \right] = U_1 V_{u1}$$

For maximum utilization  $V_2=V_{f2}$  and from inlet velocity diagram  $V_{u1}= V_1 \cos\alpha_1$ ,

$$\frac{1}{2} \left[ (V_1^2 - V_{f2}^2) + \frac{R}{(1-R)} (V_1^2 - V_{f2}^2) \right] = U_1 V_1 \cos\alpha_1$$

$$\frac{(V_1^2 - V_{f2}^2)}{2(1-R)} = U_1 V_1 \cos\alpha_1$$

$$\frac{\left(1 - \frac{V_{f2}^2}{V_1^2}\right)}{2(1-R)} = \frac{U_1}{V_1} \cos \alpha_1$$

But blade speed ratio  $\phi = \frac{U}{V_1}$

$$\left(1 - \frac{V_{f2}^2}{V_1^2}\right) = 2(1-R)\phi \cos \alpha_1$$

Or,

$$\frac{V_{f2}^2}{V_1^2} = 1 - 2(1-R)\phi \cos \alpha_1 \quad (3.11a)$$

Utilization factor is given by:

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

For maximum utilization  $V_2 = V_{f2}$ ,

$$\epsilon_{max} = \frac{V_1^2 - V_{f2}^2}{V_1^2 - RV_{f2}^2}$$

$$\epsilon_{max} = \frac{1 - \frac{V_{f2}^2}{V_1^2}}{1 - R \frac{V_{f2}^2}{V_1^2}}$$

From equation (3.11a)

$$\epsilon_{max} = \frac{1 - [1 - 2(1-R)\phi \cos \alpha_1]}{1 - R[1 - 2(1-R)\phi \cos \alpha_1]}$$

$$\epsilon_{max} = \frac{2(1-R)\phi \cos \alpha_1}{(1-R) + 2\phi R(1-R)\cos \alpha_1}$$

$$\epsilon_{max} = \frac{2\phi \cos \alpha_1}{1 + 2\phi R \cos \alpha_1} \quad (3.11b)$$

### 3.4 General Analysis of Turbines:

Power generating turbomachines are generally referred to as turbines. Turbines may run with compressible fluids like air or steam or with incompressible fluids like water. The quantity of interest in the power generating device is the work output. These machines are divided into axial, radial and mixed flow devices depending on the flow direction in the rotor blades.

**3.4.1 Axial Flow Turbines:** Axial flow machine are those in which the fluid enters and leaves the rotor at the same radius as shown in figure 3.5. Hence, for axial flow turbines  $U_1 = U_2$ . In these kinds of machines, the flow velocity ( $V_f$  or  $V_a$ ) is assumed to be constant from inlet to outlet. Axial flow turbines comprise the familiar steam turbines, gas turbines etc.

Energy transfer for axial flow turbine is:

$$e = \frac{1}{2} [(V_1^2 - V_2^2) - (V_{r1}^2 - V_{r2}^2)]$$

Degree of reaction for axial flow turbine is:

$$R = \frac{[(V_{r2}^2 - V_{r1}^2)]}{[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)]}$$

Utilization factor for axial flow turbine is:

$$\epsilon = \frac{[(V_1^2 - V_2^2) - (V_{r1}^2 - V_{r2}^2)]}{[V_1^2 - (V_{r1}^2 - V_{r2}^2)]}$$

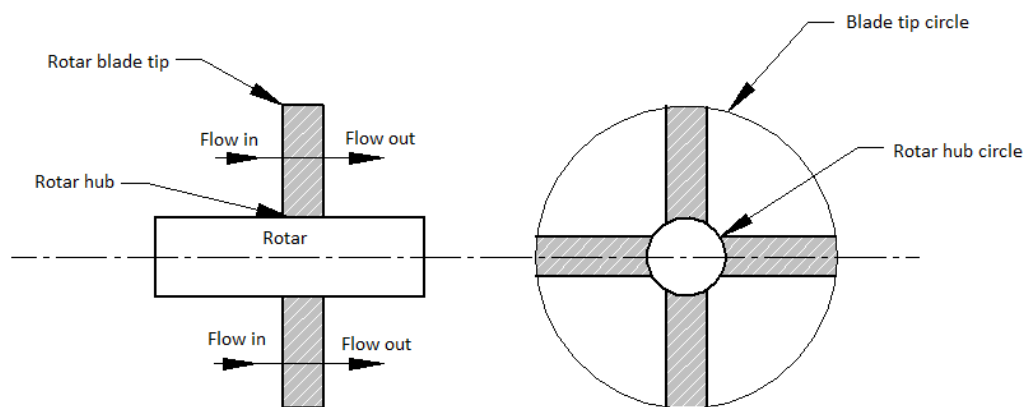


Fig. 3.5 Axial flow turbine

**Question No 3.8(a): Explain why turbines with reaction  $R > 1$  and  $R < 0$  are not in practical use?**  
(VTU, Dec-10)

**Answer:** Degree of reaction can be given as:

$$R = \frac{\text{Change in Static Pressure}}{\text{Change in Total Pressure}} = \frac{\text{Change in Static Pressure}}{\text{Change in Dynamic pressure} + \text{Change in Static Pressure}}$$

If  $R > 1$ ,

$$\frac{\text{Change in Static Pressure}}{\text{Change in Total Pressure}} > 1$$

Or,  $\text{Change in Static Pressure} > \text{Change in Total Pressure}$ , this is not practically possible.

Therefore turbine with reaction  $R > 1$  is not in practical use.

Degree of reaction can also be given as:

$$R = \frac{\text{Change in Total Pressure} - \text{Change in Dynamic Pressure}}{\text{Change in Total Pressure}}$$

If  $R < 0$

$$\frac{\text{Change in Total Pressure} - \text{Change in Dynamic Pressure}}{\text{Change in Total Pressure}} < 0$$

$$(Change\ in\ Total\ Pressure - Change\ in\ Dynamic\ Pressure) < 0$$

Or,  $Change\ in\ Total\ Pressure < Change\ in\ Dynamic\ Pressure$ , this is not practically possible. Therefore turbine with reaction  $R < 0$  is not in practical use.

**3.4.1.1 Velocity Diagrams:**

**Question No 3.8(b): Sketch velocity diagrams for  $R=0$ ,  $R=0.5$  and  $R=1$  and label. (VTU, Dec-12)**

**Answer:** For impulse axial flow turbine,  $R=0$ , thus  $V_{r1}$  should be equal to  $V_{r2}$  and if the blades are equiangular then,  $\beta_1=\beta_2$  as shown in figure 3.6 (a). Here energy transfer is purely due to change in dynamic pressure.

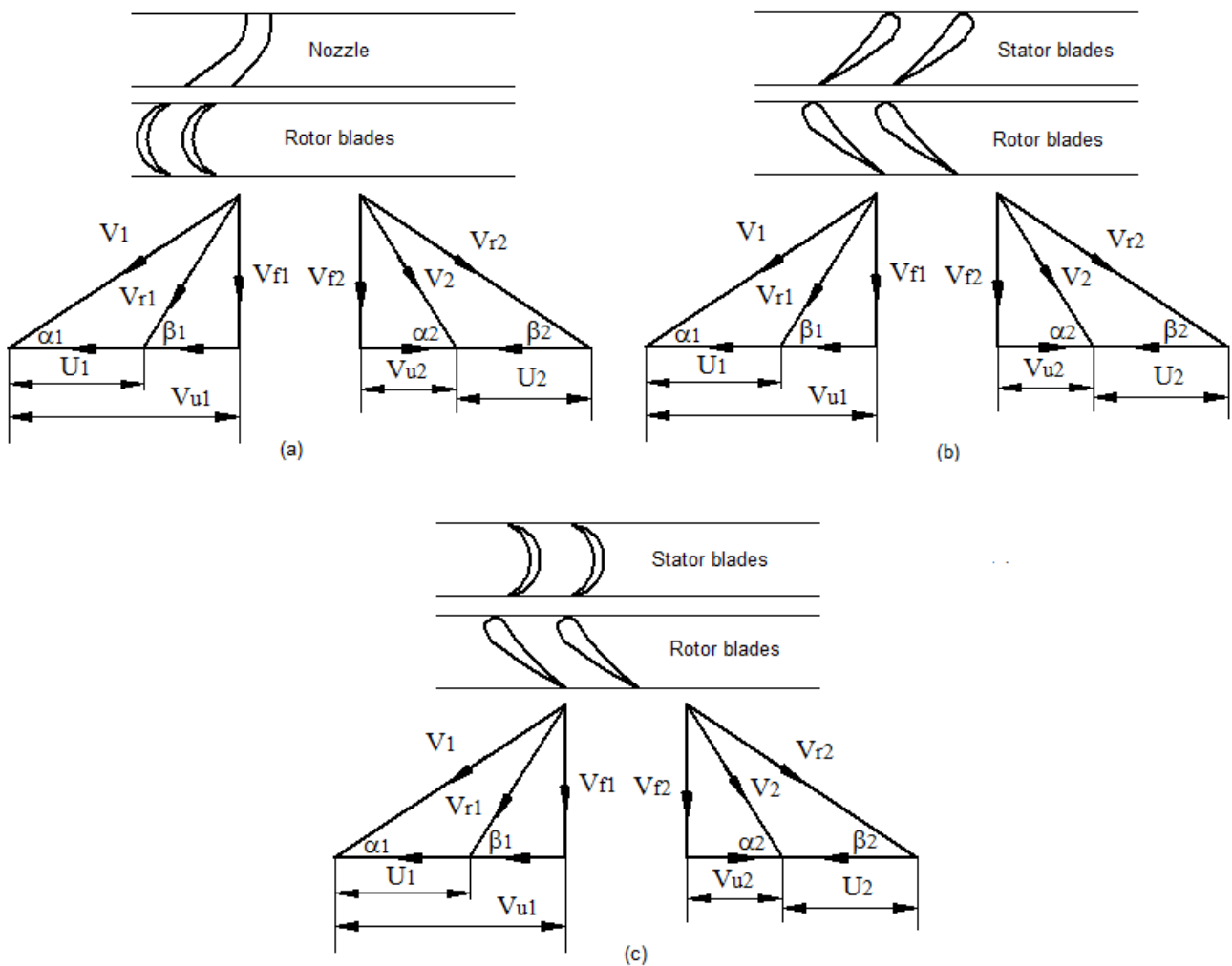


Fig. 3.6 Velocity triangles for axial flow turbine  $R = 0$ ,  $R = 0.5$  and  $R = 1$

For 50% reaction axial flow turbine,  $R=0.5$ , thus  $(V_1^2 - V_2^2) = (V_{r2}^2 - V_{r1}^2)$  and if the stator and rotor blades are symmetric (two blades are identical but orientations are different) then,  $\alpha_1=\beta_2$  and  $\alpha_2=\beta_1$  and also  $V_1=V_{r2}$  and  $V_2=V_{r1}$  as shown in figure 3.6 (b). Here energy transfer due to change in dynamic pressure is equal to energy transfer due to change in static pressure.

For fully (100%) reaction axial flow turbine,  $R=1$ , thus  $V_1$  should be equal to  $V_2$  and also  $\alpha_1=\alpha_2$  as shown in figure 3.6 (c). Here stator acts purely as a directional device and doesn't take part in the



energy conversion process. The rotor acts both as the nozzle and as the energy transfer device, so energy transfer is purely due to change in static pressure.

**3.4.1.2 Utilization Factor for R = 0 and R = 1:**

**Question No 3.9:** Derive an expression for the utilization factor for an axial flow impulse turbine stage which has equiangular rotor blades, in terms of the fixed blade angle at inlet and speed ratio and show the variation of utilization factor and speed ratio in the form of a graph. (VTU, May/June-10)

**Answer:** The velocity diagram for an axial flow impulse turbine stage with equiangular rotor blades is shown in figure 3.7.

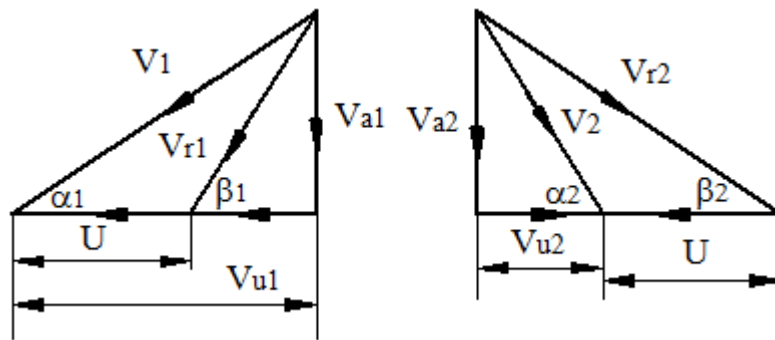


Fig. 3.7 Velocity diagram for axial flow impulse turbine.

For this machine,  $R=0$  and  $V_{r1}=V_{r2}$  and  $\beta_1=\beta_2$  (equiangular blades).

Utilization factor is given by:

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

But  $R=0$ ,

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2}$$

From outlet velocity diagram,  $V_2^2 = V_{r2}^2 + U^2 - 2UV_{r2}\cos\beta_2$

But  $V_{r1}=V_{r2}$  and  $\beta_1=\beta_2$ , then  $V_2^2 = V_{r1}^2 + U^2 - 2UV_{r1}\cos\beta_1$

From inlet velocity diagram,  $V_1^2 = V_{r1}^2 + U^2 - 2UV_{r1}\cos(180 - \beta_1)$

Or,  $V_1^2 = V_{r1}^2 + U^2 + 2UV_{r1}\cos\beta_1$

Then,  $V_1^2 - V_2^2 = 4UV_{r1}\cos\beta_1$

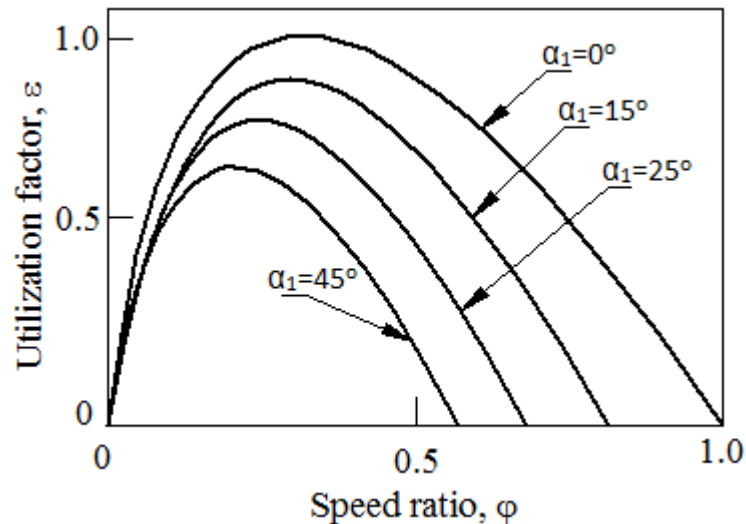


Fig.3.8 Variation of  $\epsilon$  with  $\phi$  for an impulse turbine

From inlet velocity diagram,  $\cos\beta_1 = \frac{(V_{u1}-U)}{V_{r1}}$  and  $\cos\alpha_1 = \frac{V_{u1}}{V_1}$

Then,  $V_1^2 - V_2^2 = 4U(V_{u1} - U)$

Or,  $V_1^2 - V_2^2 = 4U(V_1 \cos\alpha_1 - U)$

Then,

$$\epsilon = \frac{4U(V_1 \cos\alpha_1 - U)}{V_1^2}$$

But  $\phi = \frac{U}{V_1}$ , then

$$\epsilon = 4\phi(\cos\alpha_1 - \phi)$$

This means that utilization factor ( $\epsilon$ ) varies parabolically with the speed ratio ( $\phi$ ) and is zero both at  $\phi=0$  and at  $\phi=\cos\alpha_1$ . The variation of  $\epsilon$  with  $\phi$  is as shown in figure 3.8.

**Question No 3.10:** Derive an expression for the utilization factor for an fully reaction axial flow turbine stage, in terms of the fixed blade angle at inlet and speed ratio and show the variation of utilization factor and speed ratio in the form of a graph.

**Answer:** Figure 3.7 gives the velocity diagram for the axial flow turbine. For fully reaction axial flow turbine,  $R=1$  and  $V_1=V_2$  and also  $\alpha_1=\alpha_2$ .

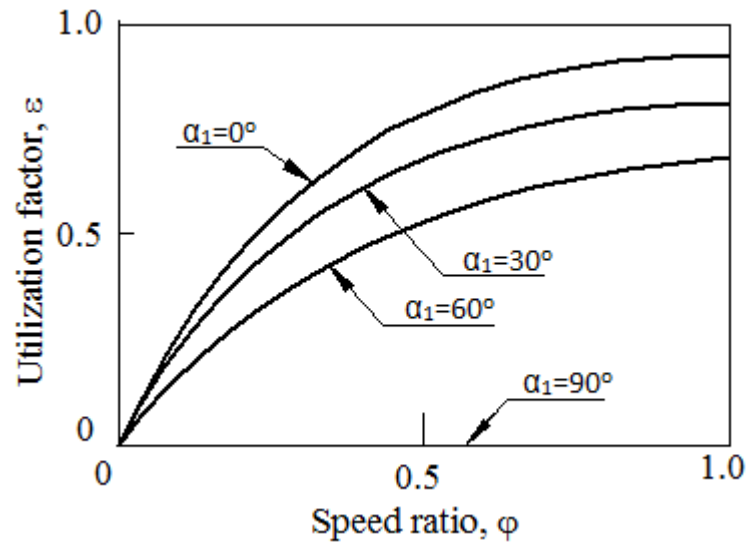


Fig.3.9 Variation of  $\epsilon$  with  $\phi$  for an fully reaction turbine

Utilization factor is given by:

$$\epsilon = \frac{e}{e + \frac{V_2^2}{2}} = \frac{U(V_{u1} + V_{u2})}{U(V_{u1} + V_{u2}) + \frac{V_1^2}{2}}$$

From inlet velocity diagram,  $\cos\alpha_1 = \frac{V_{u1}}{V_1} \Rightarrow V_{u1} = V_1 \cos\alpha_1$

And, from outlet velocity diagram,  $\cos\alpha_2 = \frac{V_{u2}}{V_2} \Rightarrow V_{u2} = V_2 \cos\alpha_2 = V_1 \cos\alpha_1$

Then,

$$\epsilon = \frac{2UV_1 \cos\alpha_1}{2UV_1 \cos\alpha_1 + \frac{V_1^2}{2}} = \frac{1}{1 + \frac{1}{4U \cos\alpha_1}}$$

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*Assistant Professor*  
*Department of Mechanical Engineering,*  
*BMS Institute of Technology and Management.*

Or,

$$\epsilon = \frac{1}{1 + \frac{1}{4\phi \cos\alpha_1}}$$

The variation of  $\epsilon$  with  $\phi$  for fully reaction axial flow turbine stage is as shown in figure 3.9. When  $\alpha_1=90^\circ$ , the utilization factor becomes zero irrespective of the speed ratio.

**3.4.1.3 Variation of Maximum Utilization Factor with Nozzle (Stator) Angle:** The general velocity diagram of axial flow turbine for maximum utilization is as shown if figure 3.10.

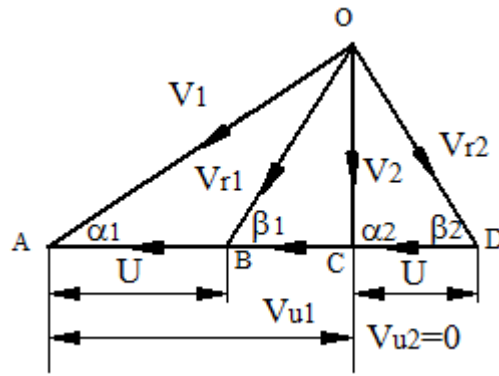


Fig. 3.10 General velocity diagram for maximum utilization (Common apex method)

Utilization factor is given by:

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

From triangle OAC,  $\sin\alpha_1 = \frac{OC}{OA} = \frac{V_2}{V_1} \Rightarrow V_2 = V_1 \sin\alpha_1$

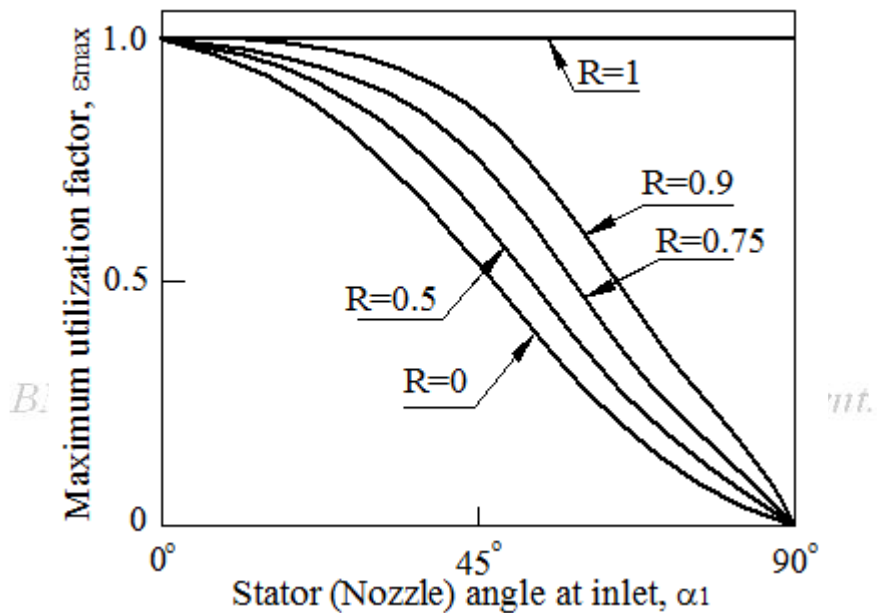


Fig. 3.11 Variation of  $\epsilon_{max}$  with  $\alpha_1$  in an axial flow turbine stage

Then,

$$\epsilon_{max} = \frac{V_1^2 - V_1^2 \sin^2\alpha_1}{V_1^2 - RV_1^2 \sin^2\alpha_1} = \frac{V_1^2(1 - \sin^2\alpha_1)}{V_1^2(1 - R\sin^2\alpha_1)}$$

$$\epsilon_{max} = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$$

For an axial flow impulse turbine  $R=0$ , then

$$\epsilon_{max} = \cos^2\alpha_1 \tag{3.12}$$

For a 50% reaction axial flow turbine  $R=0.5$ , then

$$\epsilon_{max} = \frac{\cos^2 \alpha_1}{1 - 0.5 \sin^2 \alpha_1} \tag{3.13}$$

The variation of  $\epsilon_{max}$  with  $\alpha_1$ , using  $R$  as a parameter is exhibited in figure 3.11, for all values of  $R$ ,  $\epsilon_{max}$  is unity when  $\alpha_1=0$  and becomes zero when  $\alpha_1=90^\circ$ .

**3.4.1.4 Zero-angle Turbine:** When  $\alpha_1=0$  and if the requirements for maximum utilization are maintained ( $V_2 = V_1 \sin \alpha_1 = 0$ ), the velocity diagram collapse into a straight line, results in Zero-angle turbine. The shape of the rotor blade which theoretically achieves  $\epsilon_{max}=1$  is shown in figure 3.12. Evidently the blade is semi-cylindrical in shape, with a turning angle of  $180^\circ$ . This turbine cannot work in practice, since a finite velocity  $V_2$  with an axial component is necessary to produce a steady flow at the wheel exit. However this shows that the nozzle angle should be as small as possible. The turbine that has a rotor-bucket with its shape approximately semi-cylindrical is the Pelton wheel, a hydraulic turbine. Even here, the bucket turns the water through  $165^\circ$  instead  $180^\circ$  so that the utilization factor is never unity in any real turbine.

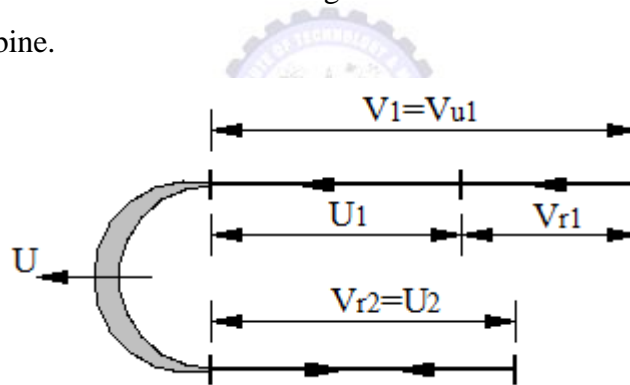


Fig.3.12 Shape of blade needed to produce  $\epsilon_{max}=1$

**3.4.1.5 Optimum Blade Speed Ratio for  $R = 0$  and  $R = 0.5$ :**

**Question No 3.11:** Draw the velocity diagram of an axial flow impulse turbine for maximum utilization and show that the optimum blade speed ratio for an axial flow impulse turbine is

$\phi_{opt} = \frac{\cos \alpha_1}{2}$ , where  $\alpha_1$  is the nozzle angle at inlet. Or

For an axial flow impulse turbine obtain the condition for maximum utilization factor.

(VTU, Jul/Aug-02)

**Answer:** The velocity diagram of an axial flow impulse turbine for maximum utilization is as shown in figure 3.13. For an axial flow impulse turbine  $V_{r1}=V_{r2}$  and  $\beta_1=\beta_2$ .

From triangle OAC,

$$\cos \alpha_1 = \frac{AC}{OA} = \frac{AB + BC}{OA}$$

Velocity triangles OBC and OCD are congruent, hence  $BC=CD=U$

Then,

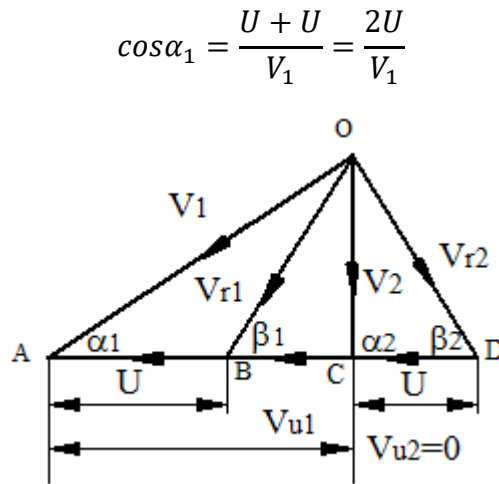


Fig. 3.13 Velocity diagram of an axial flow impulse turbine for maximum utilization

But blade speed ratio  $\phi = \frac{U}{V_1}$

Then,

$$\cos\alpha_1 = 2\phi$$

Or, the optimum blade speed ratio

$$\phi_{opt} = \frac{\cos\alpha_1}{2} \tag{3.14}$$

The optimum blade speed ratio is the blade speed ratio at which utilization factor will be the maximum.

From equation (3.11b),

$$\epsilon_{max} = \frac{2\phi \cos\alpha_1}{1 + 2\phi R \cos\alpha_1}$$

Substitute for axial flow impulse turbine  $R=0$  and for maximum utilization condition  $\phi = \frac{\cos\alpha_1}{2}$ , then

$$\epsilon_{max} = \cos^2\alpha_1$$

The above equation same as equation (3.12)

**Note:** Congruent triangles are exactly the same, that is their side lengths are the same and their interior/exterior angles are also same. We can create congruent triangles by rotating, translating or reflecting the original.

Similar triangles look the same but their side lengths are proportional to each other and their interior/exterior angles are same. We can create similar triangles by dilating the original figure (in other words making it smaller or larger by a scale factor).

**Question No 3.12:** Draw the velocity diagram of an axial flow 50% reaction turbine for maximum utilization and show that the optimum blade speed ratio for an axial flow 50% reaction turbine is  $\phi_{opt} = \cos\alpha_1$ , where  $\alpha_1$  is the nozzle angle at inlet.

**Answer:** For an axial flow 50% reaction turbine  $V_1=V_{r2}$  and  $V_2=V_{r1}$  and also  $\alpha_1=\beta_2$  and  $\alpha_2=\beta_1$ . The velocity diagram of this turbine for maximum utilization is as shown in figure 3.14.

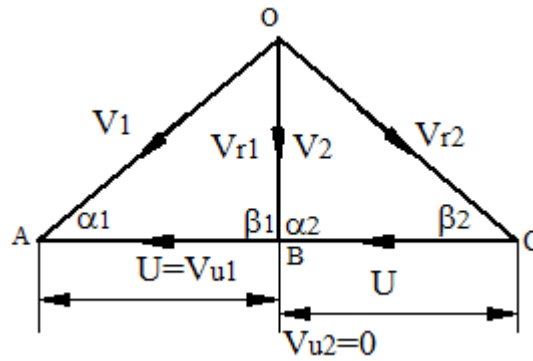


Fig. 3.14 Velocity diagram of an axial flow 50% reaction turbine for maximum utilization

From triangle OAB,

$$\cos\alpha_1 = \frac{AB}{OA} = \frac{U}{V_1}$$

But blade speed ratio  $\phi = \frac{U}{V_1}$

Then,

$$\cos\alpha_1 = \phi$$

Or, the optimum blade speed ratio

$$\phi_{opt} = \cos\alpha_1 \tag{3.15}$$

The optimum blade speed ratio is the blade speed ratio at which utilization factor will be the maximum.

From equation (3.11b),

$$\epsilon_{max} = \frac{2\phi \cos\alpha_1}{1 + 2\phi R \cos\alpha_1}$$

Substitute for a 50% reaction axial flow turbine  $R=0.5$  and for maximum utilization condition  $\phi = \cos\alpha_1$ , then

$$\begin{aligned} \epsilon_{max} &= \frac{2\cos^2\alpha_1}{1 + 2(0.5)\cos^2\alpha_1} \\ \epsilon_{max} &= \frac{\cos^2\alpha_1}{\frac{1}{2}[1 + 2(0.5)\cos^2\alpha_1]} \\ \epsilon_{max} &= \frac{\cos^2\alpha_1}{\frac{1}{2}[1 + 2(0.5)(1 - \sin^2\alpha_1)]} \\ \epsilon_{max} &= \frac{\cos^2\alpha_1}{1 - 0.5 \sin^2\alpha_1} \end{aligned}$$

The above equation same as equation (3.13)

### 3.4.1.6 Comparison between Impulse Turbine and 50% Reaction Turbine:

**Question No 3.13:** Show that for maximum utilization the work output per stage of an axial flow impulse machine (with equiangular rotor blades) is double that of a 50% reaction stage which has the same blade speed. Assume that axial velocity remains constant for both machines.

(VTU, Dec-08/Jan-09)

**Answer:** Let  $U_I$  and  $U_R$  be the blade speed of an axial flow impulse turbine and 50% reaction turbine respectively.

Work output per stage or energy transfer per stage by impulse turbine is given by,

$$e_I = U_I(V_{u1} - V_{u2})$$

For maximum utilization factor,  $V_{u2} = 0$

Then,  $e_I = U_I V_{u1}$

From impulse turbine velocity diagram for maximum utilization (Fig. 3.13)

$$AC = AB + BC \Rightarrow V_{u1} = U_I + U_I = 2U_I$$

Then,  $e_I = 2U_I^2$  (3.16)

Work output per stage or energy transfer per stage by impulse turbine is given by,

$$e_R = U_R V_{u1}$$

From 50% reaction turbine velocity diagram for maximum utilization (Fig. 3.14)

$$AB = V_{u1} = U_R$$

Then,  $e_R = U_R^2$  (3.17)

From equations (3.16) and (3.17), for same blade speed  $U_I = U_R$

$$e_I = 2e_R$$

For the maximum utilization the energy transfer in axial flow impulse turbine is double than that of axial flow 50% reaction turbine for the same blade speed.

**Question No 3.14:** Show that for maximum utilization and for same amount of energy transfer in an axial flow impulse turbine and axial flow reaction turbine with 50% degree of reaction  $U_R = \sqrt{2U_I^2}$ ,

where  $U_R$  and  $U_I$  are blade speeds of reaction turbine and impulse turbine respectively.

(VTU, Feb-06)

**Answer:** From equation (3.16), for axial flow impulse turbine

$$e_I = 2U_I^2$$

From equation (3.17), for axial flow 50% reaction turbine

$$e_R = U_R^2$$

For same energy transfer,  $e_R = e_I$

$$U_R^2 = 2U_I^2$$



Or,

$$U_R = \sqrt{2U_I^2}$$

**Question No 3.15:** Show that for maximum utilization and for same absolute velocity and inlet nozzle angle, the blade speed of axial flow 50% reaction turbine is double that of axial flow impulse turbine. (VTU, Jul-07)

**Answer:** From equation (3.13), optimum speed ratio for axial flow impulse turbine is

$$\varphi_{opt} = \frac{\cos\alpha_1}{2} = \frac{U_I}{V_1}$$

Or,

$$V_1 \cos\alpha_1 = 2U_I$$

From equation (3.14), optimum speed ratio for axial flow 50% reaction turbine is

$$\varphi_{opt} = \cos\alpha_1 = \frac{U_R}{V_1}$$

Or,

$$V_1 \cos\alpha_1 = U_R$$

For same absolute velocity ( $V_1$ ) and inlet nozzle angle ( $\alpha_1$ ),

$$U_R = 2U_I$$

For same absolute velocity and inlet nozzle angle, the blade speed of axial flow 50% reaction turbine is double that of axial flow impulse turbine.

**Question No 3.16:** Derive relations for maximum energy transfer and maximum utilization factor in case of axial flow impulse turbine and 50% reaction turbine. (VTU, May/Jun-10)

**Answer:** From equation (3.16), maximum energy transfer for axial flow impulse turbine is

$$e_I = 2U_I^2$$

From equation (3.17), maximum energy transfer for axial flow 50% reaction turbine is

$$e_R = U_R^2$$

From equation (3.12), maximum utilization factor for an axial flow impulse turbine is

$$\epsilon_{max} = \cos^2\alpha_1$$

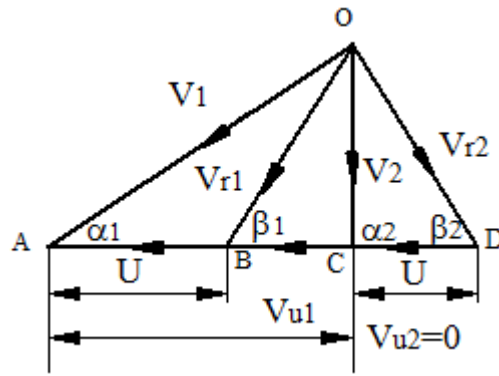
From equation (3.13), maximum utilization factor for an axial flow 50% reaction turbine is

$$\epsilon_{max} = \frac{\cos^2\alpha_1}{1-0.5\sin^2\alpha_1}$$

**Question No 3.17:** Show that maximum utilization factor of an axial flow turbine with degree of reaction  $1/4$ , the relationship of blade speed  $U$  to absolute velocity at rotor inlet  $V_1$  (speed ratio) is given by  $\varphi = \frac{U}{V_1} = \frac{2}{3} \cos\alpha_1$ , where  $\alpha_1$  is the nozzle angle with respect to tangential direction at inlet.

(VTU, Jun/Jul-09, Jun/Jul-11)

**Answer:** The velocity diagram of axial flow turbine for maximum utilization is given in figure 3.10.



For axial flow turbine, degree of reaction is:

$$R = \frac{[(V_{r2}^2 - V_{r1}^2)]}{[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)]} = \frac{1}{4}$$

$$(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2) = 4(V_{r2}^2 - V_{r1}^2)$$

Or,  $(V_1^2 - V_2^2) = 3(V_{r2}^2 - V_{r1}^2)$  (3.18)

From triangle OAC,  $\sin\alpha_1 = \frac{OC}{OA} = \frac{V_2}{V_1} \Rightarrow V_2 = V_1 \sin\alpha_1$

Or,  $V_2^2 = V_1^2 \sin^2\alpha_1$

From triangle OCD,  $V_{r2}^2 = V_2^2 + U^2$

Or,  $V_{r2}^2 = V_1^2 \sin^2\alpha_1 + U^2$

By applying cosine rule to triangle OAB,

$$V_{r1}^2 = V_1^2 + U^2 - 2UV_1 \cos\alpha_1$$

Substitute the values of  $V_2^2$ ,  $V_{r2}^2$  and  $V_{r1}^2$  in equation (3.18),

$$V_1^2 - V_1^2 \sin^2\alpha_1 = 3[V_1^2 \sin^2\alpha_1 + U^2 - (V_1^2 + U^2 - 2UV_1 \cos\alpha_1)]$$

$$4(V_1^2 - V_1^2 \sin^2\alpha_1) = 6UV_1 \cos\alpha_1$$

$$4V_1^2 \cos^2\alpha_1 = 6UV_1 \cos\alpha_1$$

Or,

$$\phi = \frac{U}{V_1} = \frac{2}{3} \cos\alpha_1$$

**3.4.2 Radial Flow Turbines:** Radial flow turbines are radial inward flow turbomachines, here fluid flows across the rotor blades radially from outer radius (tip radius) to inner radius (hub radius) of the rotor as shown in figure 3.15. Therefore radial turbines are also known as *centripetal turbomachines*. Since the fluid enters and leaves the rotor at different radius  $U_1 \neq U_2$ .

**Question No 3.18:** A radial turbomachine has no inlet whirl. The blade speed at the exit is twice that of the inlet. Radial velocity is constant throughout. Taking the inlet blade angle as  $45^\circ$ , show that energy transfer per unit mass is given by  $e = 2V_m^2(\cot\beta_2 - 2)$ , where  $\beta_2$  is the blade angle at exit with respect to tangential direction. (VTU, Jun/Jul-11)

**Answer:** The data given in the problem are:

$$V_{u1} = 0, U_2 = 2U_1, V_{m1} = V_{m2} = V_m, \beta_1 = 45^\circ (\because U_1 = V_{m1})$$

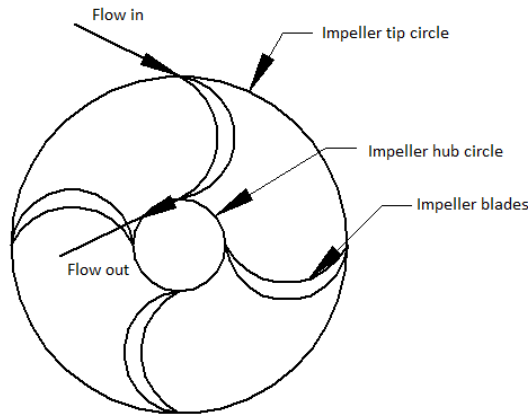
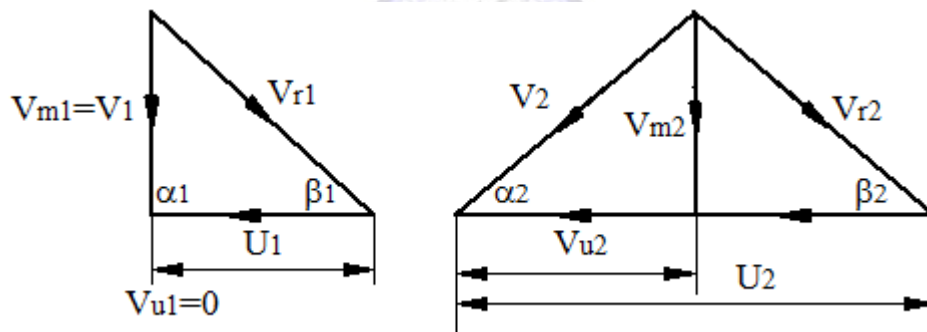


Fig. 3.15 Radial flow turbine

The velocity diagram for the above conditions is as follows



Energy transfer of general radial flow turbomachine is given by,

$$e = (U_1 V_{u1} - U_2 V_{u2})$$

But  $V_{u1} = 0$ ,

$$e = -U_2 V_{u2} \tag{3.19}$$

From outlet velocity triangle,  $V_{u2} = U_2 - x_2$

$$\text{But, } \cot \beta_2 = \frac{x_2}{V_{m2}} \Rightarrow x_2 = V_{m2} \cot \beta_2$$

Then,  $V_{u2} = U_2 - V_{m2} \cot \beta_2$

Substitute  $V_{u2}$  in equation (3.19)

$$e = -U_2 (U_2 - V_{m2} \cot \beta_2)$$

From given data,  $U_2 = 2U_1 = 2V_{m1} = 2V_{m2} = 2V_m$

$$e = -2V_m (2V_m - V_m \cot \beta_2)$$

$$e = 2V_m^2 (\cot \beta_2 - 2) \tag{3.20}$$

**Question No 3.19:** A radial turbomachine has no inlet whirl. The blade speed at the exit is twice that of the inlet. Radial velocity is constant throughout. Taking the inlet blade angle as  $45^\circ$ , show that degree of reaction is given by  $R = \frac{(2 + \cot\beta_2)}{4}$ , where  $\beta_2$  is the blade angle at exit with respect to tangential direction. (VTU, Jun/Jul-11, Dec-12, Jun/Jul-13)

**Answer:** The data given in the problem are:

$$V_{u1} = 0, U_2 = 2U_1, V_{m1} = V_{m2} = V_m, \beta_1 = 45^\circ (\because U_1 = V_{m1})$$

The velocity diagram for the above conditions is as same as the Question No 3.18.

Degree of reaction for general radial flow turbomachine is given by:

$$R = \frac{e - \frac{1}{2}(V_1^2 - V_2^2)}{e}$$

$$\text{But, } e = 2V_m^2(\cot\beta_2 - 2)$$

From inlet velocity triangle,  $V_1^2 = V_{m1}^2 = V_m^2$

By applying cosine rule to outlet velocity triangle,  $V_2^2 = U_2^2 + V_{r2}^2 - 2U_2V_{r2}\cos\beta_2$

$$\text{But, } \sin\beta_2 = \frac{V_{m2}}{V_{r2}} \Rightarrow V_{r2} = \frac{V_{m2}}{\sin\beta_2}$$

$$\text{Then, } V_2^2 = U_2^2 + \frac{V_{m2}^2}{\sin^2\beta_2} - 2U_2 \frac{V_{m2}}{\sin\beta_2} \cos\beta_2$$

From given data,  $U_2 = 2U_1 = 2V_{m1} = 2V_{m2} = 2V_m$

$$\text{Then, } V_2^2 = 4V_m^2 + V_m^2 \operatorname{cosec}^2\beta_2 - 4V_m^2 \cot\beta_2$$

$$\text{Or, } V_2^2 = 4V_m^2 + V_m^2(1 + \cot^2\beta_2) - 4V_m^2 \cot\beta_2$$

$$V_2^2 = 5V_m^2 + V_m^2 \cot^2\beta_2 - 4V_m^2 \cot\beta_2$$

Then,

$$R = \frac{2V_m^2(\cot\beta_2 - 2) - \frac{1}{2}[V_m^2 - (5V_m^2 + V_m^2 \cot^2\beta_2 - 4V_m^2 \cot\beta_2)]}{2V_m^2(\cot\beta_2 - 2)}$$

$$R = \frac{4V_m^2(\cot\beta_2 - 2) - [V_m^2 - (5V_m^2 + V_m^2 \cot^2\beta_2 - 4V_m^2 \cot\beta_2)]}{4V_m^2(\cot\beta_2 - 2)}$$

$$R = \frac{V_m^2 \cot^2\beta_2 - 4V_m^2}{4V_m^2(\cot\beta_2 - 2)}$$

$$R = \frac{2 + \cot\beta_2}{4}$$

**Question No 3.20:** Why the discharge blade angle has considerable effect in the analysis of a turbomachine? Give reasons. (VTU, Dec-10, Jun/Jul-11)

**Answer:** The energy transfer for radial flow turbomachines in terms of discharge blade angle is  $e = 2V_m^2(\cot\beta_2 - 2)$ . This equation gives that, for  $\beta_2 > 26.5^\circ$  'e' is negative and continuously

increases with  $\beta_2$ . As 'e' negative for these values of  $\beta_2$ , the machine will act as pump or compressor. For  $\beta_2 < 26.5^\circ$  'e' is positive and machine will act as a turbine.

The degree of reaction for radial flow turbomachine in terms of discharge blade angle is  $R = \frac{2 + \cot\beta_2}{4}$ . This equation gives that, for  $\beta_2$  in the range of  $26.5^\circ$  to  $153.5^\circ$ , the value of R decreases linearly from near unity to very small positive value. For  $\beta_2 = 153.5^\circ$ ,  $R=0$  and hence machine will act as impulse turbine.

The effect of discharge blade angle on energy transfer and degree of reaction of turbomachine is shown in figure 3.16.

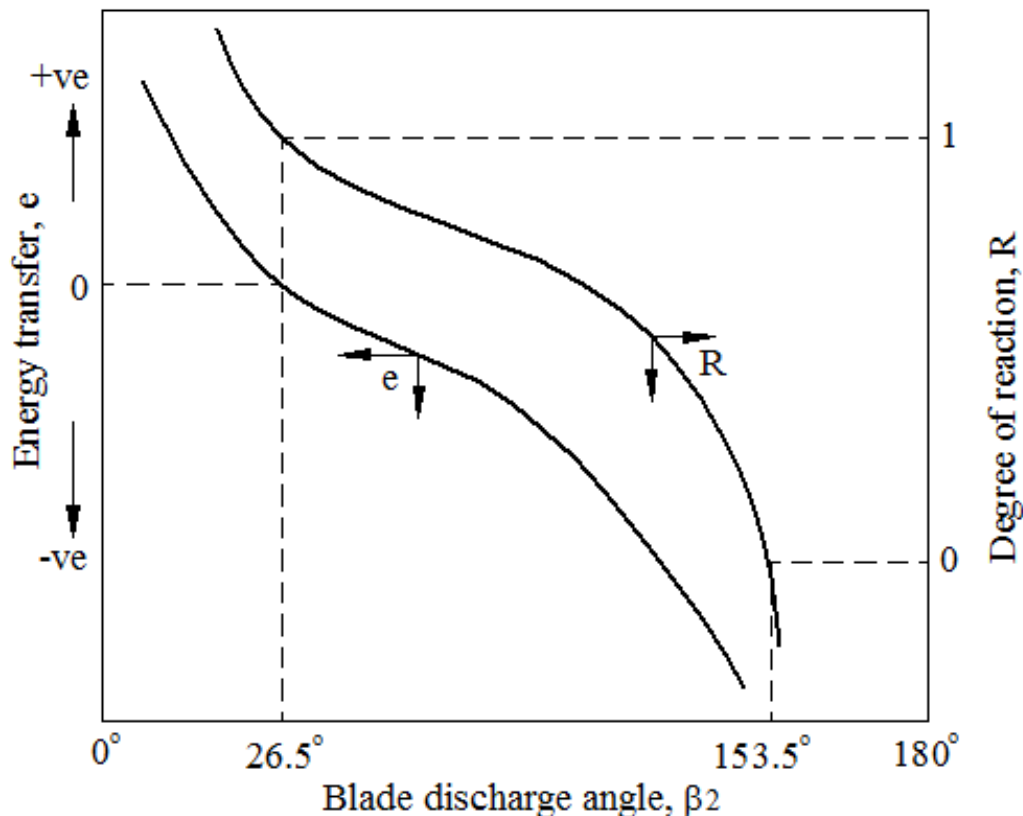


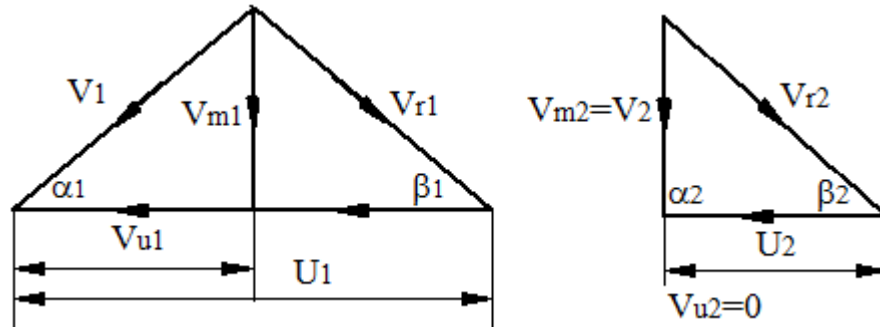
Fig. 3.16 Effect of discharge blade angle on energy transfer and degree of reaction

**Question No 3.21:** An inward flow radial reaction turbine has radial discharge at outlet with outlet blade angle is  $45^\circ$ . The radial component of absolute velocity remains constant throughout and equal to  $\sqrt{2gH}$  where  $g$  is the acceleration due to gravity and  $H$  is the constant head. The blade speed at inlet is twice that at outlet. Express the energy transfer per unit mass and the degree of reaction in terms of  $\alpha_1$ , where  $\alpha_1$  is the direction of the absolute velocity at inlet. At what value of  $\alpha_1$  will be the degree of reaction zero and unity? What are the corresponding values of energy transfer per unit mass? (VTU, Jan/Feb-06)

**Answer:** The data given in the problem are:

$$\alpha_2 = 90^\circ (\because V_{u2} = 0), \beta_2 = 45^\circ (\because U_2 = V_{m2}), V_{m1} = V_{m2} = \sqrt{2gH}, U_1 = 2U_2,$$

The velocity diagram for the above conditions is as follows



Energy transfer of inward radial flow reaction turbine is given by,

$$e = (U_1 V_{u1} - U_2 V_{u2})$$

But  $V_{u2} = 0$ ,

$$e = U_1 V_{u1}$$

From inlet velocity triangle,  $\cot \alpha_1 = \frac{V_{u1}}{V_{m1}} \Rightarrow V_{u1} = V_{m1} \cot \alpha_1$

Then,

$$e = U_1 V_{m1} \cot \alpha_1$$

From given data,  $U_1 = 2U_2 = 2V_{m2} = 2V_{m1}$

Then,

$$e = 2V_{m1}^2 \cot \alpha_1$$

From given data,  $V_{m1} = V_{m2} = \sqrt{2gH}$

Then,

$$e = 4gH \cot \alpha_1$$

Degree of reaction for inward radial flow reaction turbine is given by:

$$R = \frac{e - \frac{1}{2}(V_1^2 - V_2^2)}{e}$$

But,  $e = 4gH \cot \alpha_1$

From inlet velocity triangle,  $\sin \alpha_1 = \frac{V_{m1}}{V_1} \Rightarrow V_1 = \frac{\sqrt{2gH}}{\sin \alpha_1} \Rightarrow V_1^2 = \frac{2gH}{\sin^2 \alpha_1}$

From outlet velocity triangle,  $V_2 = V_{m2} \Rightarrow V_2^2 = V_{m2}^2 = 2gH$

Then,

$$R = \frac{4gH \cot \alpha_1 - \frac{1}{2} \left[ \frac{2gH}{\sin^2 \alpha_1} - 2gH \right]}{4gH \cot \alpha_1}$$

$$R = \frac{4 \cot \alpha_1 - \left[ \frac{1 - \sin^2 \alpha_1}{\sin^2 \alpha_1} \right]}{4 \cot \alpha_1} = \frac{4 \cot \alpha_1 - \left[ \frac{\cos^2 \alpha_1}{\sin^2 \alpha_1} \right]}{4 \cot \alpha_1}$$

$$R = \frac{4 \cot \alpha_1 - \cot^2 \alpha_1}{4 \cot \alpha_1}$$

Or,

$$R = \frac{4 - \cot \alpha_1}{4}$$

At  $\alpha_1 = 14.04^\circ$ ,  $R = 0$  then  $e = 16gH$  J/kg

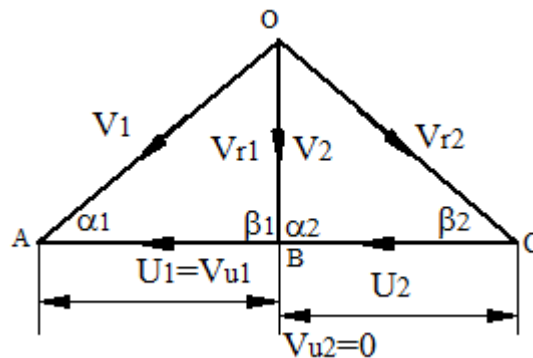
At  $\alpha_1 = 90^\circ$ ,  $R = 1$  then  $e = 0$

**Question No 3.22:** For a centripetal turbine with guide blade angle  $\alpha_1$  and radial blades at the inlet. The radial velocity is constant and there is no whirl velocity at discharge. Show that the degree of reaction is 0.5. Also derive an expression for utilization factor in terms of  $\alpha_1$ . (VTU, Jun/Jul-08, Dec-14/Jan 15)

**Answer:** The data given in the problem are:

$$\beta_1 = 90^\circ, V_{m1} = V_{m2}, V_{u2} = 0 (\because \alpha_2 = 90^\circ)$$

The velocity diagram for the above conditions is as follows



Degree of reaction for a centripetal turbine is given by:

$$R = \frac{e - \frac{1}{2}(V_1^2 - V_2^2)}{e}$$

Energy transfer of a centripetal turbine is given by,  $e = (U_1 V_{u1} - U_2 V_{u2})$

But  $V_{u2} = 0$ ,

$$e = U_1 V_{u1}$$

From inlet velocity triangle,  $V_{u1} = U_1$

Then,

$$e = U_1^2$$

From inlet velocity triangle,  $V_1^2 - V_{m1}^2 = U_1^2$

But  $V_{m1} = V_{m2} = V_2$ , then  $V_1^2 - V_2^2 = U_1^2$

Thus,

$$R = \frac{U_1^2 - \frac{1}{2}(U_1^2)}{U_1^2}$$

$$R = 0.5$$

Utilization factor for centripetal turbine is given by,

$$\epsilon = \frac{e}{e + \frac{V_2^2}{2}}$$

From inlet velocity triangle,  $\tan\alpha_1 = \frac{V_{r1}}{U_1} \Rightarrow V_{r1} = U_1 \tan\alpha_1$

But  $V_2 = V_{m2} = V_{m1} = V_{r1} = U_1 \tan\alpha_1$

Then,

$$\epsilon = \frac{U_1^2}{U_1^2 + \frac{U_1^2 \tan^2\alpha_1}{2}}$$

$$\epsilon = \frac{2}{2 + \tan^2\alpha_1}$$

Or,

$$\epsilon = \frac{2}{2 + \frac{\sin^2\alpha_1}{\cos^2\alpha_1}} = \frac{2\cos^2\alpha_1}{2\cos^2\alpha_1 + \sin^2\alpha_1}$$

$$\epsilon = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}$$

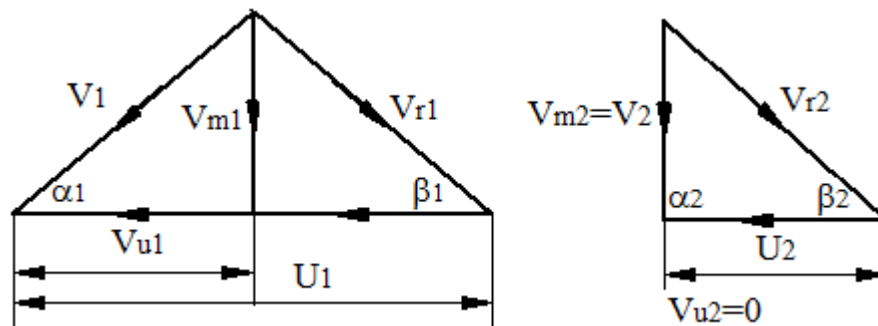
**Question No 3.23:** In an inward flow radial hydraulic turbine, degree of reaction is  $R$  and utilization factor is  $\epsilon$ . Assuming the radial velocity component is constant throughout and there is no tangential component of absolute velocity at outlet, show that the inlet nozzle angle is given by

$$\alpha_1 = \cot^{-1} \sqrt{\frac{(1-R)\epsilon}{(1-\epsilon)}} \quad (\text{VTU, Jan-04, Dec-12})$$

**Answer:** The data given in the problem are:

$$V_{m1} = V_{m2} = V_m, V_{u2} = 0 \quad (\because \alpha_2 = 90^\circ)$$

The velocity diagram for the above conditions is as follows



Utilization factor is given by:

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

From inlet velocity triangle,  $\sin\alpha_1 = \frac{V_{m1}}{V_1} \Rightarrow V_{m1} = V_1 \sin\alpha_1$



From outlet velocity triangle,  $V_2 = V_{m2} = V_{m1} = V_1 \sin \alpha_1$

Then,

$$\epsilon = \frac{V_1^2 - V_1^2 \sin^2 \alpha_1}{V_1^2 - R V_1^2 \sin^2 \alpha_1} = \frac{\cos^2 \alpha_1}{1 - R \sin^2 \alpha_1}$$

$$\epsilon = \frac{\frac{\cos^2 \alpha_1}{\sin^2 \alpha_1}}{\frac{1}{\sin^2 \alpha_1} - R} = \frac{\cot^2 \alpha_1}{\operatorname{cosec}^2 \alpha_1 - R} = \frac{\cot^2 \alpha_1}{1 + \cot^2 \alpha_1 - R}$$

$$\epsilon(1 + \cot^2 \alpha_1 - R) = \cot^2 \alpha_1$$

$$\epsilon - \epsilon R = \cot^2 \alpha_1 - \epsilon \cot^2 \alpha_1$$

Or,

$$\cot^2 \alpha_1 (1 - \epsilon) = \epsilon(1 - R)$$

$$\alpha_1 = \cot^{-1} \sqrt{\frac{(1 - R)\epsilon}{(1 - \epsilon)}}$$

### 3.5 General Analysis of Power-absorbing Turbomachines:

Compressors and pumps are power absorbing turbomachines, since they raise the stagnation pressure or enthalpy of a fluid through mechanical energy intake. The quantity of interest in the power absorbing device is the stagnation enthalpy or pressure rise of the flowing fluid due to the work. In power absorbing machines, the reference direction to define the various angles is often the axis than the tangent to the rotor-tip. Like turbines, these machines may be divided into axial, radial and mixed flow devices depending on the flow direction in the rotor blades.

**3.5.1 Axial Flow Compressors and Pumps:** In axial flow machines, the blade speed is the same at the rotor inlet and outlet. Each compressor stage consists usually of a stator and a rotor just as in a turbine. Further, there is diffuser at the exit to recover part of the exit kinetic energy of the fluid to produce an increase in static pressure. The pressure at the compressor exit will have risen due to the diffusive action in rotors and stators. If stator blades are present at the inlet they are called inlet guide-vanes. The blades at the exit section in the diffuser are called exit guide-vanes.

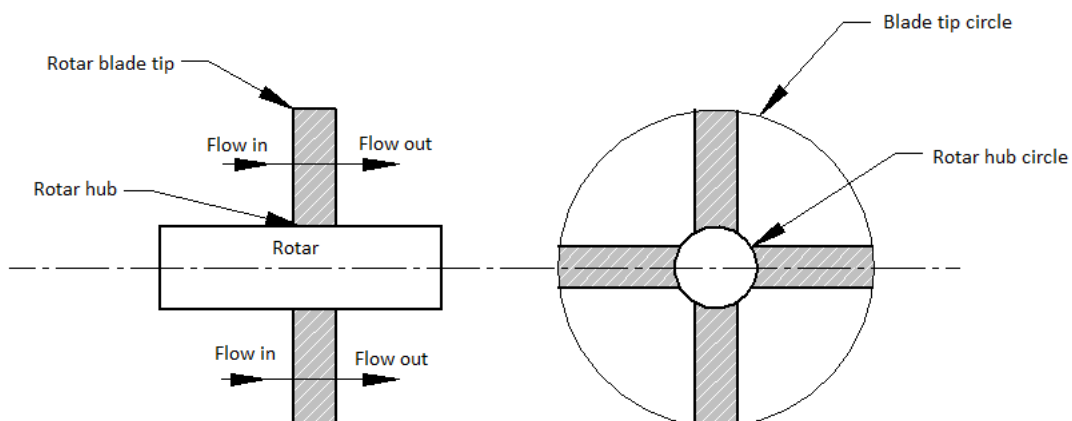


Fig. 3.17 Axial flow compressor

Energy transfer for axial flow compressor or pump is:

$$e = \frac{1}{2} [(V_2^2 - V_1^2) - (V_{r2}^2 - V_{r1}^2)]$$

Degree of reaction for axial flow compressor or pump is:

$$R = \frac{[(V_{r1}^2 - V_{r2}^2)]}{[(V_2^2 - V_1^2) - (V_{r2}^2 - V_{r1}^2)]} = \frac{e - \frac{1}{2}(V_2^2 - V_1^2)}{e}$$

**Question No 3.24:** Draw the set of velocity triangles for axial flow compressor stage and show that,  $\Delta h_o = UV_a(\tan\gamma_1 - \tan\gamma_2)$ , where  $V_a$  is axial velocity,  $U$  is blade speed and  $\gamma_1$  and  $\gamma_2$  are the inlet and outlet blade angles with respect to axial direction. Or,

Draw the set of velocity triangles for axial flow compressor stage and show that,  $\Delta h_o = UV_a \left[ \frac{\tan\beta_2 - \tan\beta_1}{\tan\beta_1 \tan\beta_2} \right]$ , where  $V_a$  is axial velocity,  $U$  is blade speed and  $\beta_1$  and  $\beta_2$  are the inlet and outlet blade angles with respect to tangential direction.

**Answer:** The general velocity diagram for axial flow compressor stage is as shown in figure 3.18. For axial flow machines the blade speed and the axial velocity may assume to be constant. That is,  $U_1 = U_2 = U$  and  $V_{a1} = V_{a2} = V_a$

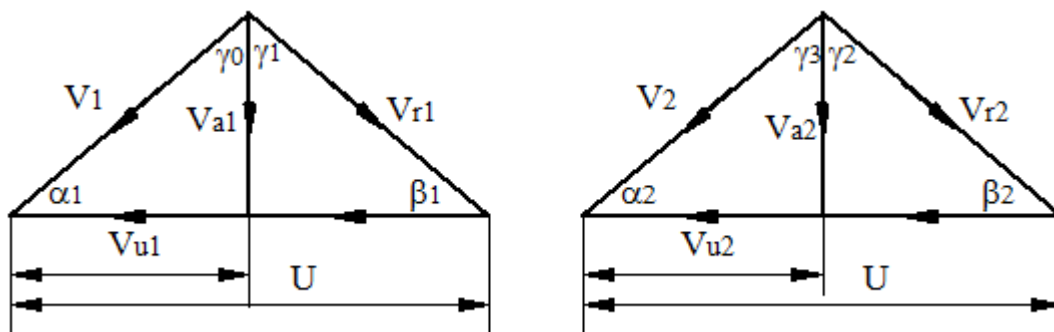


Fig. 3.18 General velocity diagram for axial flow compressor stage

Usually in an axial flow turbomachines the working fluid is either combustion gas or air. Whenever, the energy transfer occurs in these machines, then pressure energy or enthalpy of the working fluid changes. Therefore energy transfer of an axial flow compressor is given as:

$$e = \Delta h_o = U(V_{u2} - V_{u1})$$

From inlet velocity triangle,

$$\tan\gamma_o = \frac{V_{u1}}{V_{a1}} \Rightarrow V_{u1} = V_{a1} \tan\gamma_o = V_a \tan\gamma_o$$

$$\tan\gamma_1 = \frac{AB}{V_{a1}} \Rightarrow AB = V_{a1} \tan\gamma_1 = V_a \tan\gamma_1$$

$$U = AB + V_{u1} = V_a(\tan\gamma_1 + \tan\gamma_o)$$

From outlet velocity triangle,

$$\tan\gamma_3 = \frac{V_{u2}}{V_{a2}} \Rightarrow V_{u2} = V_{a2}\tan\gamma_3 = V_a\tan\gamma_3$$

$$\tan\gamma_2 = \frac{AB}{V_{a2}} \Rightarrow AB = V_{a2}\tan\gamma_2 = V_a\tan\gamma_2$$

$$U = AB + V_{u2} = V_a(\tan\gamma_2 + \tan\gamma_3)$$

Then,

$$U = V_a(\tan\gamma_1 + \tan\gamma_o) = V_a(\tan\gamma_2 + \tan\gamma_3)$$

Or,

$$(\tan\gamma_1 - \tan\gamma_2) = (\tan\gamma_3 - \tan\gamma_o)$$

Then,

$$\Delta h_o = U(V_{u2} - V_{u1}) = U(V_a\tan\gamma_3 - V_a\tan\gamma_o) = UV_a(\tan\gamma_3 - \tan\gamma_o)$$

Or,

$$\Delta h_o = UV_a(\tan\gamma_1 - \tan\gamma_2)$$

Where  $\gamma_1$  and  $\gamma_2$  are the inlet and outlet blade angles with respect to axial direction are also known as Air angles.

Or,

$$\Delta h_o = UV_a[\tan(90^\circ - \beta_1) - \tan(90^\circ - \beta_2)]$$

$$\Delta h_o = UV_a[\cot\beta_1 - \cot\beta_2]$$

Or,

$$\Delta h_o = UV_a\left[\frac{1}{\tan\beta_1} - \frac{1}{\tan\beta_2}\right]$$

$$\Delta h_o = UV_a\left[\frac{\tan\beta_2 - \tan\beta_1}{\tan\beta_1\tan\beta_2}\right]$$

Where  $\beta_1$  and  $\beta_2$  are the inlet and outlet blade angles with respect to tangential direction.

**Question No 3.25:** With the help of inlet and outlet velocity triangles, show that the degree of reaction for axial flow compressor as  $R = \frac{V_a}{U}\tan\gamma_m$ , where  $V_a$  is axial velocity,  $U$  is blade speed and  $\tan\gamma_m = \frac{\tan\gamma_1 + \tan\gamma_2}{2}$   $\gamma_1$  and  $\gamma_2$  are the inlet and outlet blade angles with respect to axial direction. (VTU, Jun-12, Dec-06/Jan-07, Jun/Jul-13) Or,

With the help of inlet and outlet velocity triangles, show that the degree of reaction for axial flow compressor as  $R = \frac{V_a}{U}\cot\beta_m$ , where  $V_a$  is axial velocity,  $U$  is blade speed and  $\cot\beta_m = \frac{\cot\beta_1 + \cot\beta_2}{2}$

$\beta_1$  and  $\beta_2$  are the inlet and outlet blade angles with respect to tangential direction. Or,

Draw the velocity triangles for an axial flow compressor and show that for an axial flow compressor having no axial thrust, the degree of reaction is given by:  $R = \frac{V_a}{2U}\left[\frac{\tan\beta_1 + \tan\beta_2}{\tan\beta_1\tan\beta_2}\right]$ , where  $V_a$  is axial velocity,  $U$  is blade speed and  $\beta_1$  and  $\beta_2$  are the inlet and outlet blade angles with respect to tangential direction. (VTU, Jan/Feb-03, Jun/Jul-11, May/Jun-10, Dec-13/Jan-14)

**Answer:** The general velocity diagram for axial flow compressor stage is as shown in figure 3.14. For axial flow machines the blade speed and the axial velocity may assume to be constant. That is,  $U_1 = U_2 = U$  and  $V_{a1} = V_{a2} = V_a$

Degree of reaction for axial flow compressor is:

$$R = \frac{\frac{1}{2}[(V_{r1}^2 - V_{r2}^2)]}{\frac{1}{2}[(V_2^2 - V_1^2) - (V_{r2}^2 - V_{r1}^2)]} = \frac{\frac{1}{2}[(V_{r1}^2 - V_{r2}^2)]}{e}$$

But,  $e = \Delta h_o = UV_a(\tan\gamma_1 - \tan\gamma_2)$

From inlet velocity triangle,  $V_{r1}^2 = AB^2 + V_{a1}^2 = V_a^2 \tan^2\gamma_1 + V_a^2$

$$V_{r1}^2 = V_a^2 + V_a^2 \tan^2\gamma_1$$

Similarly from outlet velocity triangle,

$$V_{r2}^2 = V_a^2 + V_a^2 \tan^2\gamma_2$$

Then,

$$R = \frac{V_a^2 + V_a^2 \tan^2\gamma_1 - (V_a^2 + V_a^2 \tan^2\gamma_2)}{2UV_a(\tan\gamma_1 - \tan\gamma_2)}$$

$$R = \frac{V_a(\tan^2\gamma_1 - \tan^2\gamma_2)}{2U(\tan\gamma_1 - \tan\gamma_2)}$$

$$R = \left(\frac{V_a}{U}\right) \frac{(\tan\gamma_1 + \tan\gamma_2)}{2}$$

$$R = \frac{V_a}{U} \tan\gamma_m$$

Where  $\tan\gamma_m = \frac{\tan\gamma_1 + \tan\gamma_2}{2}$   $\gamma_1$  and  $\gamma_2$  are the inlet and outlet blade angles with respect to axial direction.

Or,

$$R = \left(\frac{V_a}{U}\right) \frac{[\tan(90^\circ - \beta_1) + \tan(90^\circ - \beta_2)]}{2}$$

$$R = \left(\frac{V_a}{U}\right) \frac{[\cot\beta_1 + \cot\beta_2]}{2}$$

$$R = \frac{V_a}{U} \cot\beta_m$$

Where  $\cot\beta_m = \frac{\cot\beta_1 + \cot\beta_2}{2}$   $\beta_1$  and  $\beta_2$  are the inlet and outlet blade angles with respect to tangential direction.

Or,

$$R = \left(\frac{V_a}{2U}\right) \left[ \frac{1}{\tan\beta_1} + \frac{1}{\tan\beta_2} \right]$$

$$R = \frac{V_a}{2U} \left[ \frac{\tan\beta_1 + \tan\beta_2}{\tan\beta_1 \tan\beta_2} \right]$$

**3.5.2 Radial Flow Compressors and Pumps:** Radial flow compressors and pumps are radial outward flow turbomachines, here fluid flows across the rotor blades radially from inner radius (hub radius) to

outer radius (tip radius) of the rotor as shown in figure 3.19. Therefore radial compressors and pumps are also known as *centrifugal turbomachines*. Since the fluid enters and leaves the rotor at different radius  $U_1 \neq U_2$ . In centrifugal compressor or pump usually the absolute velocity at the entry has no tangential component, i.e.,  $V_{u1} = 0$ .

**Question No 3.26: Derive a theoretical head capacity (H-Q) relationship for centrifugal pumps and compressors and explain the influence of outlet blade angle. (VTU, Jul/Aug-05, Dec-11, Jun/Jul-14)**

**Answer:** The velocity diagram for centrifugal pumps and compressor with  $V_{u1} = 0$  is as shown in figure 3.20. Usually in a radial flow turbomachines the working fluid is either water or oil. Whenever, the energy transfer occurs in these machines, then pressure energy or potential energy of the working fluid changes.

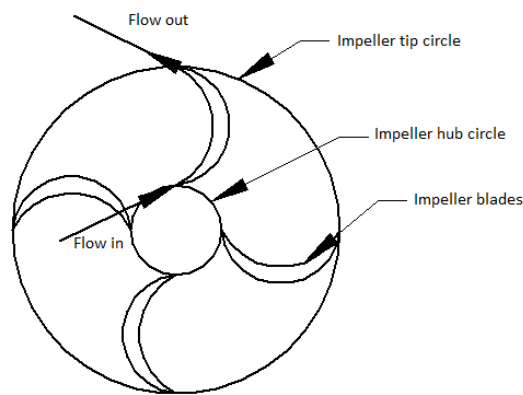


Fig. 3.19 Radial flow compressor or pump

The energy transfer of a centrifugal compressor and pump is given as:

$$e = gH = U_2 V_{u2} - U_1 V_{u1}$$

Or,  $gH = U_2 V_{u2}$  (Because,  $V_{u1} = 0$ )

From outlet velocity triangle,  $V_{u2} = U_2 - x_2$

$$\text{But, } \cot \beta_2 = \frac{x_2}{V_{m2}} \Rightarrow x_2 = V_{m2} \cot \beta_2$$

Then,

$$V_{u2} = U_2 - V_{m2} \cot \beta_2$$

Therefore,  $gH = U_2 (U_2 - V_{m2} \cot \beta_2)$

Or,

$$H = \frac{U_2^2}{g} - \frac{U_2 V_{m2}}{g} \cot \beta_2$$

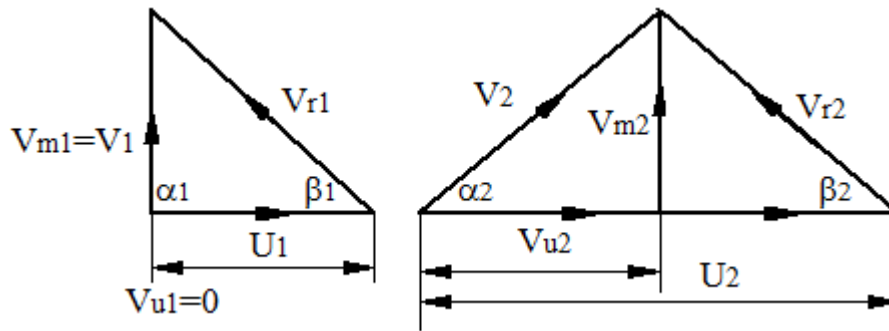


Fig. 3.20 Velocity diagram for centrifugal pumps and compressor with  $V_{u1} = 0$

Discharge at outer radius of centrifugal machine = Area of flow  $\times$  Flow velocity

$$Q = \pi D_2 B_2 \times V_{m2}$$

$$V_{m2} = \frac{Q}{\pi D_2 B_2}$$

Then,

$$H = \frac{U_2^2}{g} - \left(\frac{U_2}{g}\right) \left(\frac{Q}{\pi D_2 B_2}\right) \cot \beta_2$$

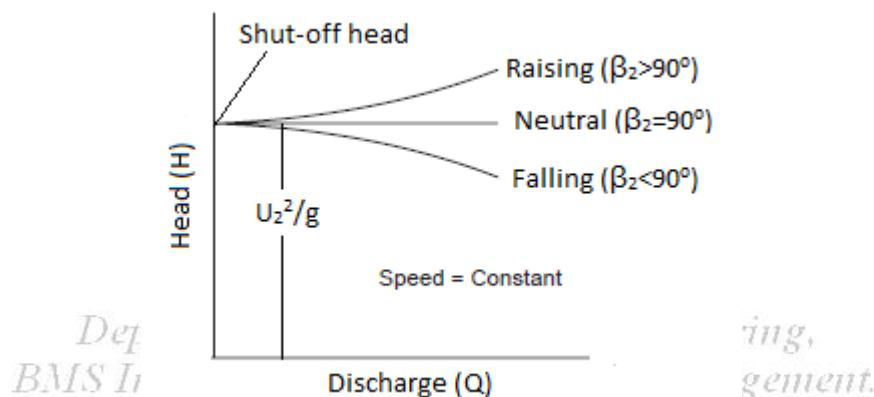


Fig. 3.21 H-Q characteristic curve for centrifugal machines

By using above equation, H-Q characteristic curve of a given impeller exit blade angle  $\beta_2$  for different values of discharge is drawn in figure 3.21.

**Question No 3.27:** Draw the inlet and outlet velocity triangles for a radial flow power absorbing turbomachines with (i) Backward curved vane (ii) Radial vane (iii) Forward curved vane. Assume inlet whirl velocity to be zero. Draw and explain the head-capacity relations for the above 3 types of vanes. (VTU, Dec-08/Jan-09, Dec-12)

**Answer:** There are three types of vane shapes in centrifugal machines namely, (i) Backward curved vane (ii) Radial vane (iii) Forward curved vane.

The vane is said to be backward curved if the angle between the rotor blade-tip and the tangent to the rotor at the exit is acute ( $\beta_2 < 90^\circ$ ). If it is a right angle ( $\beta_2 = 90^\circ$ ) the blade said to be radial and if it is greater than  $90^\circ$ , the blade is said to be forward curved. Here the blade angles measured with respect

to direction of rotor (clockwise direction). The velocity triangles at the outlet of centrifugal machines are shown in figure 3.21.

The head-capacity characteristic curve for the above 3 types of vanes is given in figure 3.16, if  $\beta_2$  lies between 0 to  $90^\circ$  (backward curved vanes),  $\cot\beta_2$  in H-Q relation is always positive. So for backward curved vanes the head developed by the machine falls with increasing discharge. For values of  $\beta_2$  between  $90^\circ$  to  $180^\circ$ ,  $\cot\beta_2$  in H-Q relation is negative. So for forward curved vanes the head developed by the machine continuously rise with increasing discharge. For  $\beta_2=90^\circ$  (radial vanes), the head is independent of flow rates and is remains constant. For centrifugal machines usually the absolute velocity at the entry has no tangential component (i.e.,  $V_{u1}=0$ ), thus the inlet velocity triangle for all the 3 types of vanes is same.

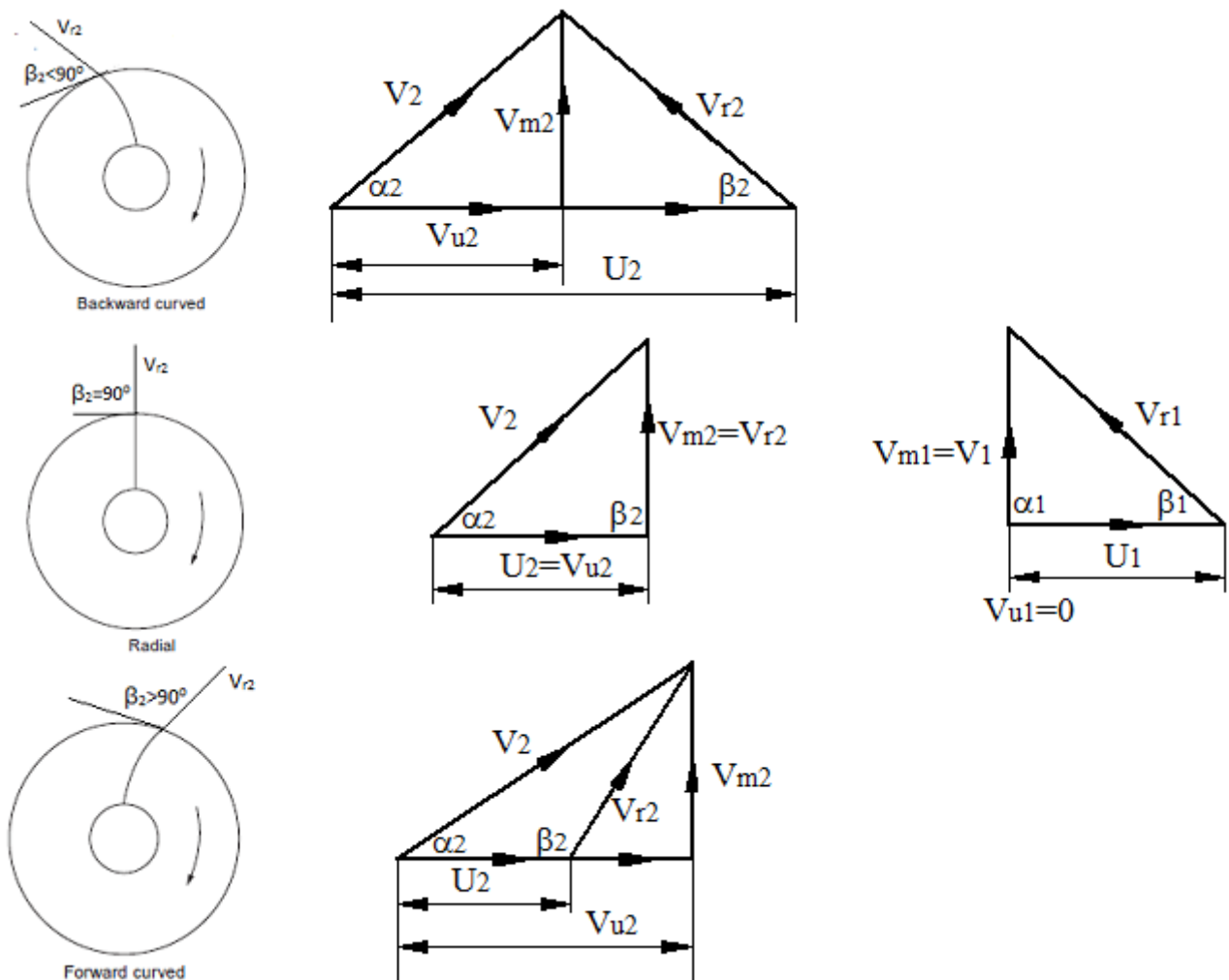


Fig. 3.21 Types of centrifugal vanes

**Question No 3.27:** Draw the velocity diagram for a power absorbing radial flow turbomachine and

show that  $R = \frac{1}{2} \left[ 1 + \frac{V_{m2} \cot \beta_2}{U_2} \right]$ . (VTU, Dec-14/Jan-15)

## Chapter 4

### STEAM TURBINES

#### 4.1 Introduction:

Steam and gas turbines are power generating machines in which the pressure energy of the fluid is converted into mechanical energy. This conversion is due to the dynamic action of fluid flowing over the blades. These blades are mounted on the periphery of a rotating wheel in the radial direction. Today the steam turbine stands as one of the most important prime movers for power generation. It converts thermal energy into mechanical work by expanding high pressure and high temperature steam. The thermal efficiency of steam turbine is fairly high compared to steam engine. The uniform speed of steam turbine at wide loads makes it suitable for coupling it with generators, centrifugal pumps, centrifugal gas compressors, etc.

#### 4.2 Classification of Steam Turbines:

Based on the action of steam on blades, steam turbines are classified into impulse turbines and reaction turbines (or impulse reaction turbines).

**4.2.1 Impulse Steam Turbine:** Impulse or impetus means sudden tendency of action without reflexes. A single-stage impulse turbine consists of a set of nozzles and moving blades as shown in figure 4.1. High pressure steam at boiler pressure enters the nozzle and expands to low condenser pressure in the nozzle. Thus, the pressure energy is converted into kinetic energy increasing the velocity of steam. The high velocity steam is then directed on a series of blades where the kinetic energy is absorbed and converted into an impulse force by changing the direction of flow of steam which gives rise to a change in momentum and therefore to a force. This causes the motion of blades. The velocity of steam decreases as it flows over the blades but the pressure remains constant, i.e. the pressure at the outlet side of the blade is equal to that at the inlet side. Such a turbine is termed as impulse turbine. Examples: De-Laval, Curtis, Moore, Zoelly, Rateau etc.



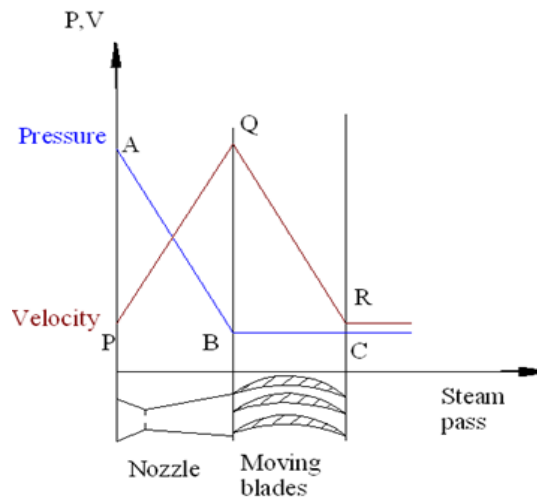


Fig. 4.1 Impulse turbine

**4.2.2 Impulse Reaction Steam Turbine:** In the impulse reaction turbine, power is generated by the combination of impulse action and reaction by expanding the steam in both fixed blades (act as nozzles) and moving blades as shown in figure 4.2. Here the pressure of the steam drops partially in fixed blades and partially in moving blades. Steam enters the fixed row of blades, undergoes a small drop in pressure and increases in velocity. Then steam enters the moving row of blades, undergoes a change in direction and momentum (impulse action), and a small drop in pressure too (reaction), giving rise to increase in kinetic energy. Hence, such a turbine is termed as impulse reaction turbine. Examples: Parson, Ljungstrom etc.

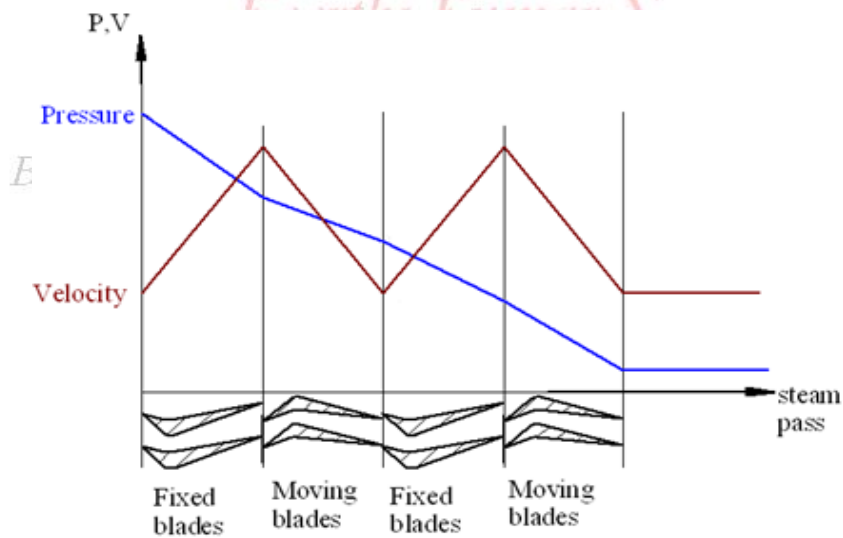


Fig. 4.2 Impulse reaction turbine

**4.3 Difference between Impulse and Reaction Turbines:**

The differences between impulse and reaction turbines are as follows:

Impulse Turbine	Reaction Turbine
✓ Complete expansion of the steam take place in	✓ Partial expansion of the steam takes place in

the nozzle, hence steam is ejected with very high kinetic energy.	the fixed blade (acts as nozzle) and further expansion takes place in the rotor blades.
✓ Blades are symmetrical in shape.	✓ Blades are non-symmetrical in shape, i.e. aerofoil section.
✓ Pressure remains constant between the ends of the moving blade. Hence relative velocity remains constant i.e., $V_{r1} = V_{r2}$	✓ Pressure drops from inlet to outlet of the moving blade. Hence relative velocity increases from inlet to outlet i.e., $V_{r2} > V_{r1}$
✓ Steam velocity at the inlet of machine is very high, hence needs compounding.	✓ Steam velocity at the inlet of machine is moderate or low, hence doesn't need compounding.
✓ Blade efficiency is comparatively low.	✓ Blade efficiency is high.
✓ Few number of stages required for given pressure drop or power output, hence machine is compact.	✓ More number of stages required for given pressure drop or power output, hence machine is bulky.
✓ Used for small power generation.	✓ Used for medium and large power generation.
✓ Suitable, where the efficiency is not a matter of fact.	✓ Suitable, where the efficiency is a matter of fact.

#### 4.4 Need for Compounding of Steam Turbines:

**Question No 4.1:** What is the need for compounding in steam turbines? Discuss any two methods of compounding. (VTU, Jul/Aug-05, Dec-06/Jan-07, Dec-09/Jan-10, Dec-13/Jan-14)

**Answer:** If the steam pressure drops from boiler pressure to condenser pressure in a single stage, exit velocity of steam from the nozzle will become very high and the turbine speed will be of the order of 30,000 rpm or more. As turbine speed is proportional to steam velocity, the carryover loss or leaving loss will be more (10% to 12%). Due to this very high speed, centrifugal stresses are developed on the turbine blades resulting in blade failure. In order to overcome all these difficulties it is necessary to reduce the turbine speed by the method of compounding. *Compounding is the method of reducing blade speed for a given overall pressure drop.*

#### 4.5 Methods of Compounding of Steam Turbine:

**Question No 4.2:** What are the various methods of compounding of steam turbines? Explain any one of them. (VTU, Jul/Aug-02)

**Answer:** Following are the methods of compounding of steam turbines:

1. Velocity compounding

2. Pressure compounding
3. Pressure and velocity compounding

#### 4.5.1 Velocity Compounding:

**Question No 4.3:** Explain with the help of a schematic diagram a two row velocity compounded turbine stage. (VTU, Jan/Feb-05, Jul-06, Dec-12)

**Answer:** A simple velocity compounded impulse turbine is shown in figure 4.3. It consists of a set of nozzles and a rotating wheel fitted with two or more rows of moving blades. One row of fixed blades is fitted between the rows of moving blades. The function of the fixed blade is to direct the steam coming from the first row of moving blades to the next row of moving blades without appreciable change in velocity.

Steam from the boiler expands completely in the nozzle, hence whole of the pressure energy converts into kinetic energy. The kinetic energy of steam gained in the nozzle is successively absorbed by rows of moving blades and steam is exited from the last row axially with very low velocity. Due to this, the rotor speed decreases considerably. The velocity compounded impulse turbine is also called the Curtis turbine stage.

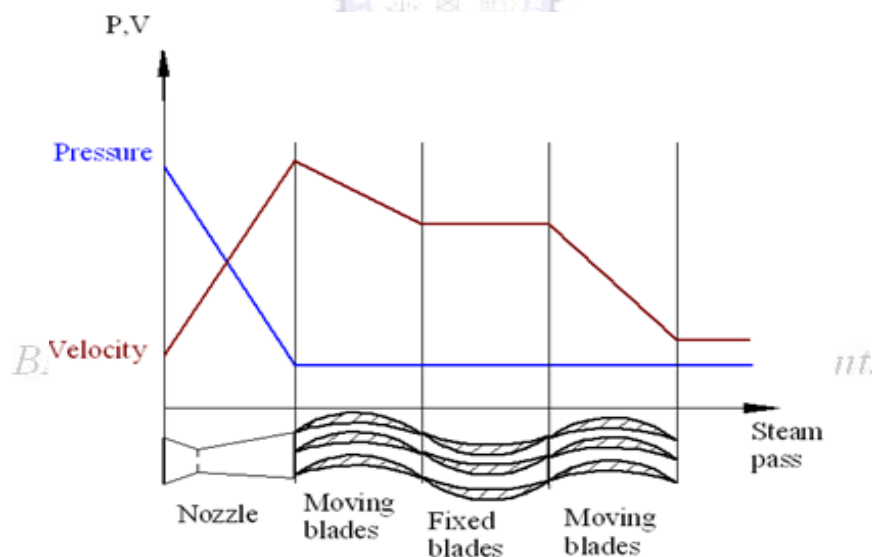


Fig. 4.3 Two stage velocity compounded impulse turbine.

#### 4.5.2 Pressure Compounding:

**Question No 4.4:** Explain briefly a two stage pressure compounded impulse turbine and show the pressure and velocity variations across the turbine. (VTU, Jul-07, May/Jun-10, Dec-12)

**Answer:** If a number of simple impulse stages arranged in series is known as pressure compounding. The arrangement contains one set of nozzles (fixed blades) at the entry of each row of moving blades. The total pressure drop doesn't take place in the first row of nozzles, but divided equally between all the nozzles as shown in figure 4.4.

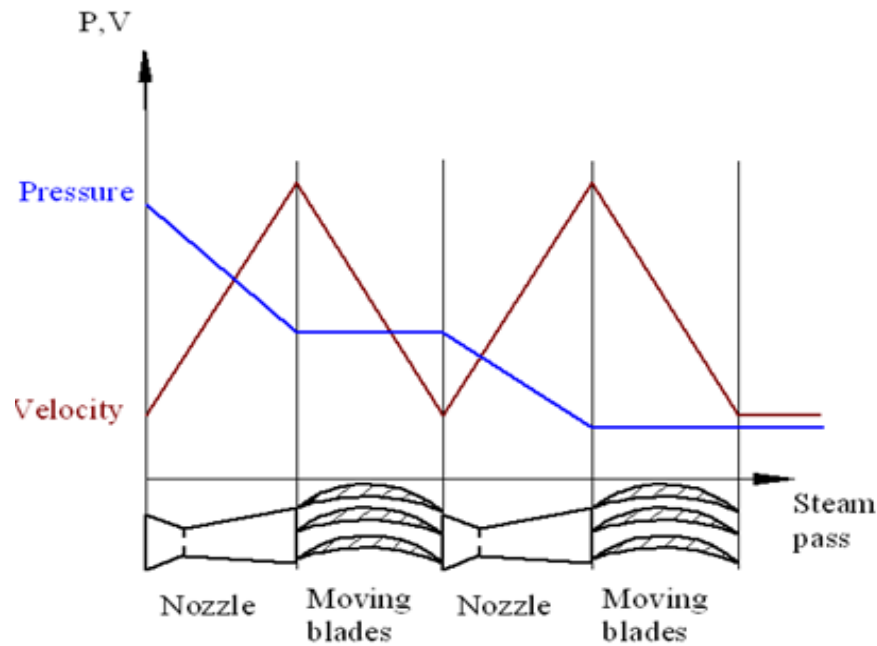


Fig. 4.4 Two stage pressure compounded impulse turbine.

The steam from the boiler is passed through the first set of nozzles in which it is partially expanded. Steam then passes over the first row of moving blades where almost all its velocity is absorbed. This completes expansion of steam in one stage. In the next stage, steam again enters the second set of nozzles and partially expands and enters the moving blades. Again the steam velocity is absorbed. This process continues till steam reaches the condenser pressure. Due to pressure compounding, smaller transformation of heat energy into kinetic energy takes place. Hence steam velocities become much lower and rotor speed decrease considerably. The pressure compounded impulse turbine is also called the Rateau turbine stage.

#### 4.5.3 Pressure-Velocity Compounding:

**Question No 4.5:** Explain with a neat sketch pressure-velocity compounding. (Dec-06/Jan-07, Jun/Jul-13)

**Answer:** If pressure and velocity are both compounded using two or more number of stages by having a series arrangement of simple velocity compounded turbines on the same shaft, it is known as pressure-velocity compounding. In this type of turbine both pressure compounding and velocity compounding methods are used. The total pressure drop of the steam is dividing into two stages and the velocity obtained in each stage is also compounded. Pressure drop occurs only in nozzles and remains constant in moving and fixed blades. As pressure drop is large in each stage only a few stages are necessary. This makes the turbine more compact than the other two types. Pressure-velocity compounding is used in Curtis turbine.

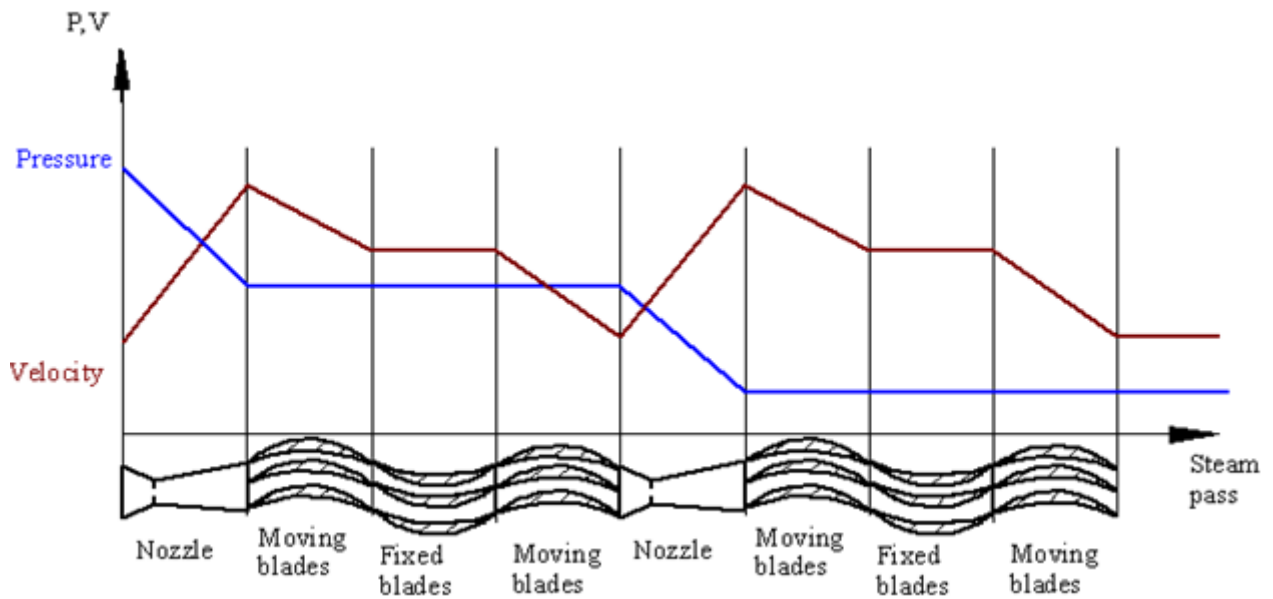


Fig. 4.5 Pressure-velocity compounded impulse turbine.

### 4.6 Efficiencies of Steam Turbine:

**Question No 4.6:** Define and explain (i) blade coefficient (ii) nozzle efficiency (iii) diagram efficiency (iv) stage efficiency. (VTU, Dec-11, Dec-12)

**Answer:** Some performance parameters of steam turbines are as follows:

(i) **Blade coefficient:** It is also known as nozzle velocity coefficient. The losses in the flow over blades are due to friction, leakage and turbulence. Blade coefficient is the ratio of the velocity at the exit to the velocity at the inlet of the blade. i.e.,

$$C_b = \frac{V_{r2}}{V_{r1}} = \frac{V_2}{V_1}$$

(ii) **Nozzle efficiency:** It is defined as the ratio of actual enthalpy change per kg of steam to the isentropic enthalpy change per kg of steam. i.e.,

$$\eta_n = \frac{\Delta h}{\Delta h'}$$

For impulse turbine,

$$\eta_n = \frac{\frac{1}{2} V_1^2}{\Delta h'}$$

For reaction turbine the stator efficiency is,

$$\eta_p = \frac{\frac{1}{2} V_1^2 - \frac{1}{2} (V_{r1}^2 - V_{r2}^2)}{\Delta h'}$$

(iii) **Diagram efficiency:** It is also known as blade efficiency or rotor efficiency. It is defined as the ratio of work done per kg of steam by the rotor to the energy available at the inlet per kg of steam. i.e.,

$$\eta_b = \frac{w}{e_a} = \frac{U\Delta V_u}{e_a}$$

For impulse turbine,  $e_a = \frac{1}{2}V_1^2$

For reaction turbine,  $e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)$

**(iv) Stage efficiency:** It is defined as the ratio of work done per kg of steam by the rotor to the isentropic enthalpy change per kg of steam in the nozzle. i.e.,

$$\eta_s = \frac{w}{\Delta h'}$$

For impulse turbine,

$$\eta_s = \frac{U\Delta V_u}{\frac{1}{2}V_1^2} \times \frac{\frac{1}{2}V_1^2}{\Delta h'}$$

Or,

$$\eta_s = \eta_b \times \eta_n$$

For reaction turbine,

$$\eta_s = \frac{U\Delta V_u}{\frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)} \times \frac{\frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)}{\Delta h'}$$

Or,

$$\eta_s = \eta_b \times \eta_p$$

### 4.7 De' Laval Turbine (Single Stage Axial Flow Impulse Turbine):

**Question No 4.7:** Show that for a single stage axial flow impulse turbine the rotor efficiency is given by,  $\eta_b = 2(\phi \cos \alpha_1 - \phi^2) \left[ 1 + C_b \frac{\cos \beta_2}{\cos \beta_1} \right]$ , where  $C_b = \frac{V_{r2}}{V_{r1}}$ ,  $\phi$  is speed ratio,  $\beta_1$  and  $\beta_2$  are rotating blade angles at inlet and exit,  $V_{r1}$  and  $V_{r2}$  are relative velocities at inlet and exit.

(VTU, Feb-06, Jun/Jul-14)

**Answer:** The combined velocity diagram for an axial flow impulse turbine is as shown in figure 4.6.

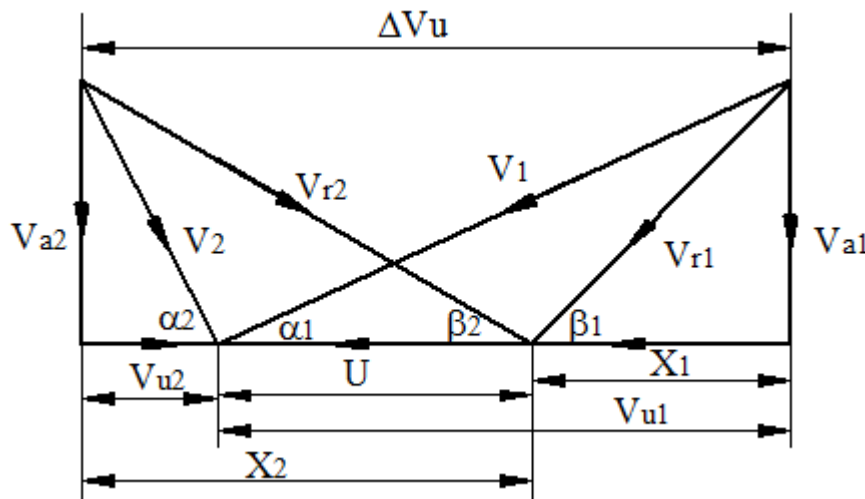


Fig. 4.6 Combined velocity diagram for an axial flow impulse turbine

Energy transfer for an axial flow turbine,

$$e = w = U\Delta V_u = U(V_{u1} + V_{u2})$$

From velocity diagram,  $V_{u1} + V_{u2} = x_1 + U + x_2 - U = x_1 + x_2$

Or,

$$V_{u1} + V_{u2} = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2$$

$$V_{u1} + V_{u2} = V_{r1}\cos\beta_1 \left[ 1 + \frac{V_{r2}\cos\beta_2}{V_{r1}\cos\beta_1} \right] = x_1 \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

Where  $C_b = \frac{V_{r2}}{V_{r1}}$ , blade velocity coefficient

$$V_{u1} + V_{u2} = (V_{u1} - U) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right] = (V_1\cos\alpha_1 - U) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

Then,

$$w = U(V_1\cos\alpha_1 - U) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

Blade or rotor efficiency is given by,

$$\eta_b = \frac{w}{e_a} = \frac{U(V_1\cos\alpha_1 - U) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]}{\frac{1}{2}V_1^2}$$

$$\eta_b = 2 \left( \frac{U}{V_1} \cos\alpha_1 - \frac{U^2}{V_1^2} \right) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

$$\eta_b = 2(\varphi\cos\alpha_1 - \varphi^2) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

Where  $\varphi = \frac{U}{V_1}$ , blade speed ratio

**Question No 4.8:** Find the condition of maximum blade efficiency in a single stage impulse turbine. (VTU, Jan/Feb-03)

**Answer:** The blade efficiency for single stage impulse turbine is given by,

$$\eta_b = 2(\varphi\cos\alpha_1 - \varphi^2) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

The variation of blade efficiency vs. speed ratio is shown in figure 4.7.

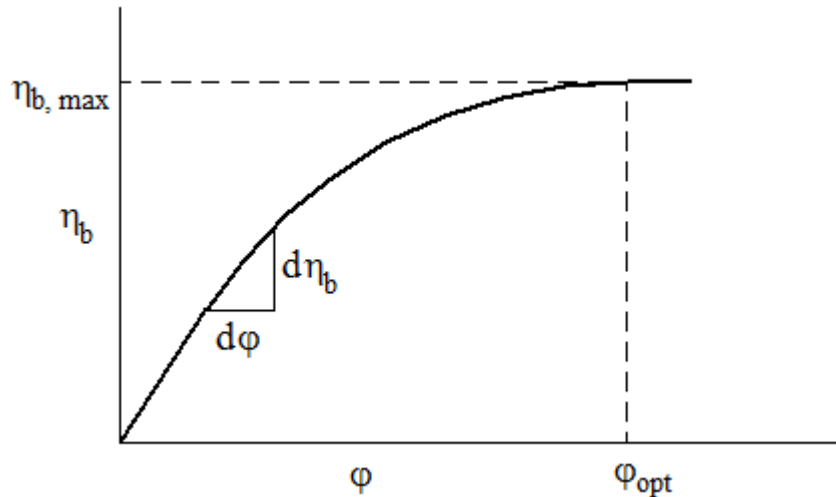


Fig. 4.7 Variation of blade efficiency vs. speed ratio

The slope for maximum blade efficiency is,

$$\frac{d\eta_b}{d\phi} = 0$$

$$\frac{d}{d\phi} \left\{ 2(\phi \cos\alpha_1 - \phi^2) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right] \right\} = 0$$

$$2(\cos\alpha_1 - 2\phi) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right] = 0$$

$$\phi_{opt} = \frac{\cos\alpha_1}{2}$$

The optimum speed ratio is the speed ratio at which the blade efficiency is the maximum.

**Question No 4.9:** For a single stage impulse turbine, prove that the maximum blade efficiency is given by  $\eta_{b,max} = \frac{\cos^2\alpha_1}{2} \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$ , where  $C_b = \frac{V_{r2}}{V_{r1}}$ ,  $\alpha_1$  is speed ratio,  $\beta_1$  and  $\beta_2$  are rotating blade angles at inlet and exit,  $V_{r1}$  and  $V_{r2}$  are relative velocities at inlet and exit. (VTU, Dec-08/Jan-09)

**Answer:** The blade efficiency for single stage impulse turbine is given by,

$$\eta_b = 2(\phi \cos\alpha_1 - \phi^2) \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

When  $\phi = \frac{\cos\alpha_1}{2}$ , the blade efficiency is the maximum, therefore

$$\eta_{b,max} = 2 \left[ \left( \frac{\cos\alpha_1}{2} \right) \cos\alpha_1 - \left( \frac{\cos\alpha_1}{2} \right)^2 \right] \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

$$\eta_{b,max} = \frac{\cos^2\alpha_1}{2} \left[ 1 + C_b \frac{\cos\beta_2}{\cos\beta_1} \right]$$

**Question No 4.10:** Prove that the maximum blade efficiency for a single stage impulse turbine with equiangular rotor blades is given by  $\eta_{b,max} = \frac{\cos^2\alpha_1}{2} [1 + C_b]$ , where  $\alpha_1$  is the nozzle angle and  $C_b$  is blade velocity coefficient. (VTU, Dec-10) Or,



*Prove that the maximum blade efficiency for a single stage impulse turbine with equiangular rotor blades is given by  $\eta_{b,max} = \cos^2 \alpha_1$ , where  $\alpha_1$  is the nozzle angle. (VTU, Jun/Jul-09, Jun/Jul-13)*

**Answer:** The maximum blade efficiency for a single stage impulse turbine is,

$$\eta_{b,max} = \frac{\cos^2 \alpha_1}{2} \left[ 1 + C_b \frac{\cos \beta_2}{\cos \beta_1} \right]$$

For equiangular rotor blades,  $\beta_1 = \beta_2$

$$\eta_{b,max} = \frac{\cos^2 \alpha_1}{2} [1 + C_b]$$

If no losses due to friction, leakage and turbulence in the flow over blades,  $V_{r1} = V_{r2}$  (i.e.  $C_b = 1$ )

$$\eta_{b,max} = \frac{\cos^2 \alpha_1}{2} [1 + 1]$$

$$\eta_{b,max} = \cos^2 \alpha_1$$

Above equation conclude that, if the flow over blades doesn't have any losses due to friction, leakage and turbulence then for a single stage impulse turbine with equiangular rotor blades maximum blade efficiency is same as maximum utilization factor.

#### **4.8 Curtis Turbine (Velocity Compounded Axial Flow Impulse Turbine):**

The velocity diagrams for first and second stages of a Curtis turbine (velocity compounded impulse turbine) are as shown in figure 4.8. The tangential speed of blade for both the rows is same since all the moving blades are mounted on the same shaft. Assume equiangular stator and rotor blades and blade velocity coefficients for stator and rotors are same.

The work done by first row of moving blades is,

$$w_1 = U \Delta V_{u1} = U(V_{u1} + V_{u2})$$

From first stage velocity diagram,

$$V_{u1} = V_1 \cos \alpha_1$$

And also,

$$V_{u2} = x_2 - U = V_{r2} \cos \beta_2 - U$$

$$V_{u2} = C_b V_{r1} \cos \beta_1 - U$$

$$(Because \beta_1 = \beta_2 \text{ and } V_{r2} = C_b V_{r1})$$

$$V_{u2} = C_b x_1 - U = C_b (V_{u1} - U) - U$$

$$(Because V_{r1} \cos \beta_1 = x_1 = V_{u1} - U)$$

$$V_{u2} = C_b (V_1 \cos \alpha_1 - U) - U$$

$$(Because V_{u1} = V_1 \cos \alpha_1)$$

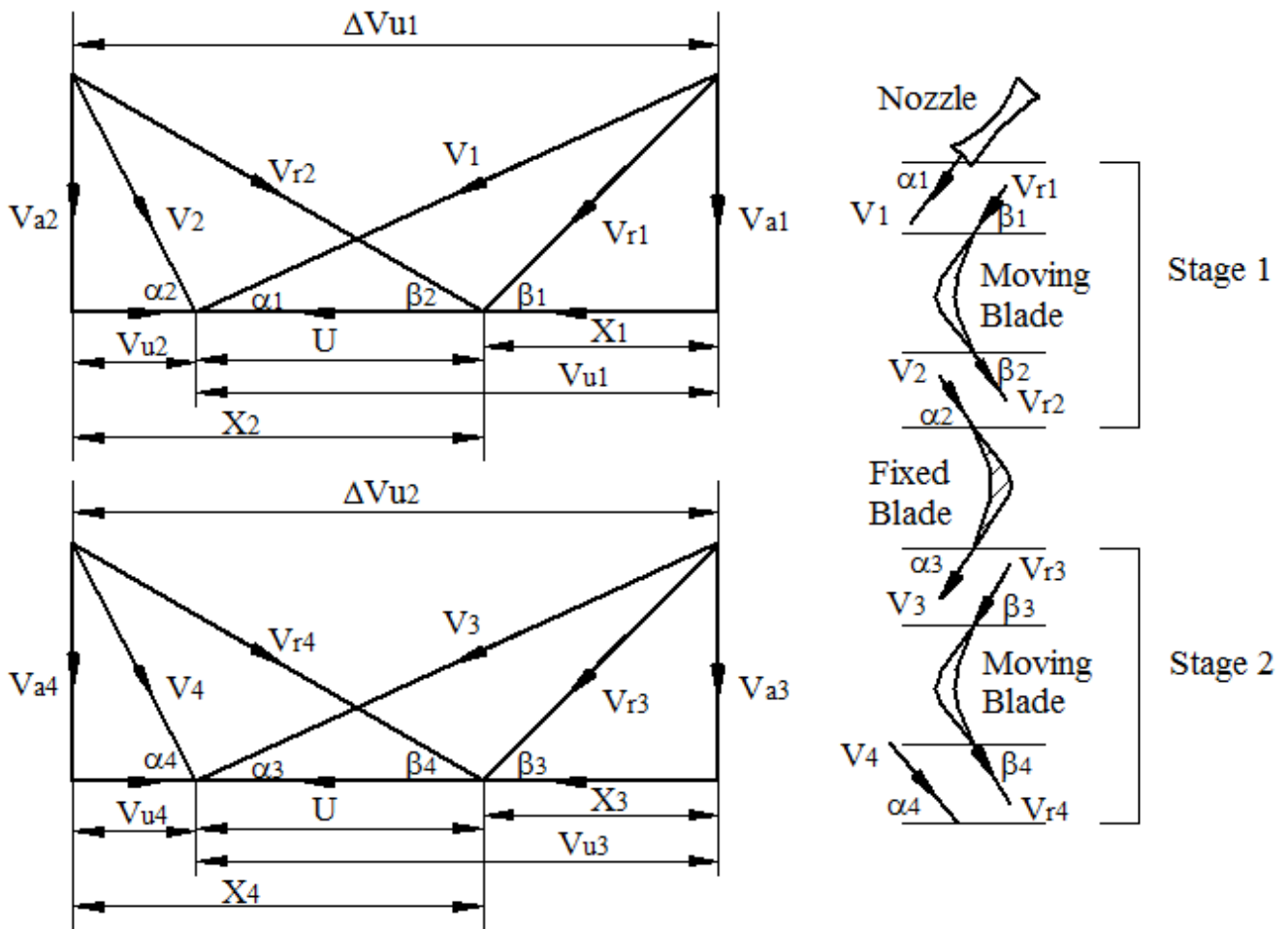


Fig. 4.8 Velocity diagrams for first and second stages of a Curtis turbine.

Then,

$$w_1 = U(V_1 \cos \alpha_1 + C_b(V_1 \cos \alpha_1 - U) - U)$$

$$w_1 = UV_1 \cos \alpha_1 + C_b UV_1 \cos \alpha_1 - C_b U^2 - U^2$$

$$w_1 = (1 + C_b)UV_1 \cos \alpha_1 - (1 + C_b)U^2$$

$$w_1 = (1 + C_b)[UV_1 \cos \alpha_1 - U^2]$$

Similarly work done by second stage is,

$$w_2 = (1 + C_b)[UV_3 \cos \alpha_3 - U^2]$$

$$w_2 = (1 + C_b)[C_b UV_2 \cos \alpha_2 - U^2]$$

(Because  $\alpha_3 = \alpha_2$  and  $V_3 = C_b V_2$ )

$$w_2 = (1 + C_b)[C_b UV_{u2} - U^2]$$

(Because  $V_2 \cos \alpha_2 = V_{u2}$ )

$$w_2 = (1 + C_b)[C_b U \{C_b(V_1 \cos \alpha_1 - U) - U\} - U^2]$$

(Because  $V_{u2} = C_b(V_1 \cos \alpha_1 - U) - U$ )

$$w_2 = (1 + C_b)[C_b^2 UV_1 \cos \alpha_1 - C_b^2 U^2 - C_b U^2 - U^2]$$

$$w_2 = (1 + C_b)[C_b^2 UV_1 \cos \alpha_1 - U^2(1 + C_b + C_b^2)]$$

The total work done by the Curtis turbine is,  $w_T = w_1 + w_2$

$$w_T = (1 + C_b)[UV_1 \cos \alpha_1 - U^2] + (1 + C_b)[C_b^2 UV_1 \cos \alpha_1 - U^2(1 + C_b + C_b^2)]$$

$$w_T = (1 + C_b)[(1 + C_b^2)UV_1 \cos \alpha_1 - U^2(2 + C_b + C_b^2)]$$

$$\text{Let, } C_b' = (1 + C_b)(1 + C_b^2) \text{ and } C_b'' = (1 + C_b)(2 + C_b + C_b^2)$$

Then,

$$w_T = [C_b' UV_1 \cos \alpha_1 - C_b'' U^2]$$

Blade or rotor efficiency is given by,

$$\eta_b = \frac{w_T}{e_a} = \frac{(C_b' UV_1 \cos \alpha_1 - C_b'' U^2)}{\frac{1}{2} V_1^2}$$

$$\eta_b = 2 \left( C_b' \left( \frac{U}{V_1} \right) \cos \alpha_1 - C_b'' \left( \frac{U^2}{V_1^2} \right) \right)$$

$$\eta_b = 2(C_b' \phi \cos \alpha_1 - C_b'' \phi^2)$$

The slope for maximum blade efficiency is,

$$\frac{d\eta_b}{d\phi} = 0$$

$$\frac{d}{d\phi} \{2(C_b' \phi \cos \alpha_1 - C_b'' \phi^2)\} = 0$$

$$2(C_b' \cos \alpha_1 - 2C_b'' \phi) = 0$$

$$\phi_{opt} = \left( \frac{C_b'}{C_b''} \right) \frac{\cos \alpha_1}{2}$$

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The maximum blade efficiency is,

$$\eta_{b,max} = 2 \left( C_b' \left( \left( \frac{C_b'}{C_b''} \right) \frac{\cos \alpha_1}{2} \right) \cos \alpha_1 - C_b'' \left( \left( \frac{C_b'}{C_b''} \right) \frac{\cos \alpha_1}{2} \right)^2 \right)$$

$$\eta_{b,max} = 2 \left( \frac{(C_b')^2}{C_b''} \left( \frac{\cos^2 \alpha_1}{2} \right) - \frac{(C_b')^2}{C_b''} \left( \frac{\cos^2 \alpha_1}{4} \right) \right)$$

$$\eta_{b,max} = \frac{(C_b')^2}{C_b''} \left( \frac{\cos^2 \alpha_1}{2} \right)$$

**Note: If blade velocity coefficient,  $C_b = 1$**

Then,  $C_b' = 4$  and  $C_b'' = 8$

For single stage impulse turbine,

$$w = 2[UV_1 \cos \alpha_1 - U^2]$$

$$\phi_{opt} = \frac{\cos \alpha_1}{2}$$

$$\eta_{b,max} = \cos^2 \alpha_1$$

For Curtis (two stage velocity compounded) turbine,

$$w_T = 4[UV_1 \cos \alpha_1 - 2U^2]$$

$$\phi_{opt} = \frac{\cos \alpha_1}{4}$$

$$\eta_{b,max} = \cos^2 \alpha_1$$

Similarly for 'n' stage Curtis (velocity compounded) turbine,

$$w = 2n[UV_1 \cos \alpha_1 - U^2]$$

$$\phi_{opt} = \frac{\cos \alpha_1}{2n}$$

$$\eta_{b,max} = \cos^2 \alpha_1$$

For all Curtis turbines the maximum blade efficiency remains same irrespective of their number of stages.

#### 4.9 Parson's Turbine (50% Axial Flow Reaction Turbine):

**Question No 4.12:** Show that for an axial flow reaction turbine, the degree of reaction is given by

$R = \left(\frac{V_a}{2U}\right) [\cot \beta_2 - \cot \beta_1]$  and also show that for axial flow 50% reaction turbine the blade speed is given by  $U = V_a [\cot \beta_2 - \cot \beta_1]$ , where  $\beta_1$  and  $\beta_2$  are inlet and outlet rotor blade angles.

Assume velocity of flow or axial velocity to be constant. (VTU, Jun-12)

**Answer:** The combined velocity diagram for an axial flow reaction turbine is as shown in figure 4.9.

From data given in the problem,  $V_{a1} = V_{a2} = V_a$

Degree of reaction for axial flow turbine,

$$R = \frac{\frac{1}{2}(V_{r2}^2 - V_{r1}^2)}{e} = \frac{(V_{r2}^2 - V_{r1}^2)}{2e}$$

From velocity diagram,  $(V_{u1} + V_{u2}) = (x_1 + U + x_2 - U)$

$$= (x_1 + x_2) = (V_{a1} \cot \beta_1 + V_{a2} \cot \beta_2)$$

Or,  $(V_{u1} + V_{u2}) = V_a (\cot \beta_1 + \cot \beta_2)$

From velocity diagram,  $\sin \beta_2 = \frac{V_{a2}}{V_{r2}} \Rightarrow V_{r2} = \frac{V_{a2}}{\sin \beta_2}$

$$V_{r2} = V_a \operatorname{cosec} \beta_2$$

Similarly,  $\sin \beta_1 = \frac{V_{a1}}{V_{r1}} \Rightarrow V_{r1} = \frac{V_{a1}}{\sin \beta_1}$

$$V_{r1} = V_a \operatorname{cosec} \beta_1$$

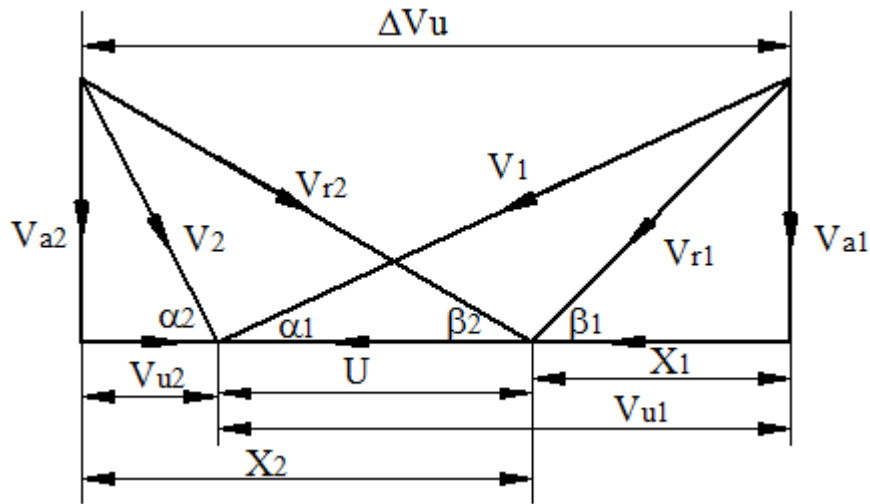


Fig. 4.9 Combined velocity diagram for an axial flow reaction turbine

Then,  $e = U(V_{u1} + V_{u2}) \Rightarrow e = UV_a(\cot\beta_1 + \cot\beta_2)$

And,  $(V_{r2}^2 - V_{r1}^2) = (V_a^2 \operatorname{cosec}^2\beta_2 - V_a^2 \operatorname{cosec}^2\beta_1) \Rightarrow (V_{r2}^2 - V_{r1}^2) = V_a^2(\operatorname{cosec}^2\beta_2 - \operatorname{cosec}^2\beta_1)$

Therefore,

$$R = \frac{V_a^2(\operatorname{cosec}^2\beta_2 - \operatorname{cosec}^2\beta_1)}{2UV_a(\cot\beta_1 + \cot\beta_2)}$$

$$R = \frac{V_a[(1 + \cot^2\beta_2) - (1 + \cot^2\beta_1)]}{2U(\cot\beta_1 + \cot\beta_2)}$$

$$R = \frac{V_a[(\cot^2\beta_2) - (\cot^2\beta_1)]}{2U(\cot\beta_1 + \cot\beta_2)}$$

$$R = \frac{V_a[(\cot\beta_2 - \cot\beta_1)(\cot\beta_2 + \cot\beta_1)]}{2U(\cot\beta_1 + \cot\beta_2)}$$

$$R = \left(\frac{V_a}{2U}\right) [\cot\beta_2 - \cot\beta_1]$$

For an axial flow 50% reaction turbine,  $R=0.5$

$$0.5 = \frac{1}{2} = \left(\frac{V_a}{2U}\right) [\cot\beta_2 - \cot\beta_1]$$

$$U = V_a[\cot\beta_2 - \cot\beta_1]$$

Alternate method:

From velocity diagram,  $U = V_{u1} - x_1 = V_{a1}\cot\alpha_1 - V_{a1}\cot\beta_1$

For an axial flow 50% reaction turbine,  $\alpha_1=\beta_2$  and  $\alpha_2=\beta_1$  and also  $V_1=V_{r2}$  and  $V_2=V_{r1}$

$$U = V_a[\cot\beta_2 - \cot\beta_1]$$

**Question No 4.13:** For a 50% reaction steam turbine, show that  $\alpha_1=\beta_2$  and  $\alpha_2=\beta_1$ , where  $\alpha_1$  and  $\beta_1$  are the inlet angles of fixed and moving blades,  $\alpha_2$  and  $\beta_2$  are the outlet angles of fixed and moving blades. (VTU, Dec-12)

**Answer:** The general equation of degree of reaction for axial flow reaction steam turbine is,

$$R = \left(\frac{V_a}{2U}\right) [\cot\beta_2 - \cot\beta_1]$$

$$R = \left(\frac{V_a}{2U}\right) [(\cot\beta_2 - \cot\alpha_1) + (\cot\alpha_1 - \cot\beta_1)]$$

From velocity diagram (fig.4.9),  $U = V_{u1} - x_1 = V_{a1}\cot\alpha_1 - V_{a1}\cot\beta_1$

Assume velocity of flow or axial velocity to be constant,  $V_{a1}=V_{a2}=V_a$

$$U = V_a(\cot\alpha_1 - \cot\beta_1)$$

Or,

$$\frac{U}{V_a} = (\cot\alpha_1 - \cot\beta_1)$$

Then,

$$R = \left(\frac{V_a}{2U}\right) \left[(\cot\beta_2 - \cot\alpha_1) + \frac{U}{V_a}\right]$$

$$R = \frac{1}{2} \left(\frac{V_a}{U}\right) (\cot\beta_2 - \cot\alpha_1) + \frac{1}{2}$$

For a 50% reaction steam turbine,  $R = 1/2$

Therefore,  $0 = \cot\beta_2 - \cot\alpha_1 \Rightarrow \cot\alpha_1 = \cot\beta_2$

Or,  $\alpha_1 = \beta_2$

From velocity diagram (fig.4.9),  $U = x_2 - V_{u2} = V_{a2}\cot\beta_2 - V_{a2}\cot\alpha_2$

Assume velocity of flow or axial velocity to be constant,  $V_{a1}=V_{a2}=V_a$

$$U = V_a(\cot\beta_2 - \cot\alpha_2)$$

Then,  $U = V_a(\cot\alpha_1 - \cot\beta_1) = V_a(\cot\beta_2 - \cot\alpha_2)$

But,  $\alpha_1 = \beta_2$

$$V_a(\cot\beta_2 - \cot\beta_1) = V_a(\cot\beta_2 - \cot\alpha_2)$$

$$\cot\beta_1 = \cot\alpha_2$$

Or,  $\alpha_2 = \beta_1$

From velocity diagram (fig.4.9),  $V_a = V_1\cos\alpha_1 = V_{r2}\cos\beta_2$

But,  $\alpha_1 = \beta_2$

$$V_1 = V_{r2}$$

$$V_a = V_2\cos\alpha_2 = V_{r1}\cos\beta_1$$

But,  $\alpha_2 = \beta_1$

$$V_2 = V_{r1}$$

These relations show that the velocity triangles at the inlet and outlet of the rotor of a 50% reaction stage are symmetrical.

**Question No 4.14:** What is meant by reaction staging? Prove that the maximum blade efficiency of Parson's (axial flow 50% reaction) turbine is given by  $\eta_{b,max} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$ .

(VTU, Jan/Feb-04, Jun/Jul-08, May/Jun-10, Dec-14/Jan-15)

**Answer:** In reaction staging the expansion of steam and enthalpy drop occurs both in fixed and moving blades. Due to the effect of continuous expansion during flow over the moving blades, the relative velocity of steam increases i.e.,  $V_{r2} > V_{r1}$ .

For Parson's (axial flow 50% reaction) turbine,  $\alpha_1 = \beta_2$  and  $\alpha_2 = \beta_1$  and also  $V_1 = V_{r2}$  and  $V_2 = V_{r1}$ , then the velocity triangles are symmetric (refer figure 4.9).

Work done by Parson's turbine,

$$w = U\Delta V_u = U(V_{u1} + V_{u2})$$

From velocity diagram,

$$w = U(V_{u1} + x_2 - U) = U(V_1\cos\alpha_1 + V_{r2}\cos\beta_2 - U)$$

But,  $\alpha_1 = \beta_2$  and  $V_1 = V_{r2}$

Then,

$$w = U(V_1\cos\alpha_1 + V_1\cos\alpha_1 - U) = 2UV_1\cos\alpha_1 - U^2$$

Or,

$$w = V_1^2 \left[ \frac{2UV_1\cos\alpha_1}{V_1^2} - \frac{U^2}{V_1^2} \right]$$

But, blade speed ratio  $\phi = \frac{U}{V_1}$

$$w = V_1^2 [2\phi\cos\alpha_1 - \phi^2]$$

For reaction turbine energy available at rotor inlet,

$$e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_{r2}^2)$$

But  $V_1 = V_{r2}$ ,

$$e_a = \frac{1}{2}V_1^2 - \frac{1}{2}(V_{r1}^2 - V_1^2) = V_1^2 - \frac{V_{r1}^2}{2}$$

From velocity diagram,

$$V_{r1}^2 = V_1^2 + U^2 - 2UV_1\cos\alpha_1 \quad (\text{By cosine rule})$$

Then,

$$e_a = V_1^2 - \frac{1}{2}[V_1^2 + U^2 - 2UV_1\cos\alpha_1]$$

$$e_a = \frac{1}{2}[V_1^2 + 2UV_1\cos\alpha_1 - U^2] = \frac{V_1^2}{2} \left[ 1 + \frac{2UV_1\cos\alpha_1}{V_1^2} - \frac{U^2}{V_1^2} \right]$$

But, blade speed ratio  $\phi = \frac{U}{V_1}$

$$e_a = \frac{V_1^2}{2} [1 + 2\phi \cos \alpha_1 - \phi^2]$$

Blade efficiency of reaction turbine,

$$\eta_b = \frac{w}{e_a} = \frac{V_1^2 [2\phi \cos \alpha_1 - \phi^2]}{\frac{V_1^2}{2} [1 + 2\phi \cos \alpha_1 - \phi^2]}$$

$$\eta_b = \frac{2[2\phi \cos \alpha_1 - \phi^2]}{[1 + 2\phi \cos \alpha_1 - \phi^2]}$$

Or,

$$\eta_b = \frac{2[1 + 2\phi \cos \alpha_1 - \phi^2] - 2}{[1 + 2\phi \cos \alpha_1 - \phi^2]}$$

$$\eta_b = 2 - \frac{2}{[1 + 2\phi \cos \alpha_1 - \phi^2]} = 2 - 2[1 + 2\phi \cos \alpha_1 - \phi^2]^{-1}$$

The slope for maximum blade efficiency is (refer figure 4.7),

$$\frac{d\eta_b}{d\phi} = 0$$

$$\frac{d}{d\phi} \{2 - 2[1 + 2\phi \cos \alpha_1 - \phi^2]^{-1}\} = 0$$

$$2[1 + 2\phi \cos \alpha_1 - \phi^2]^{-2} [2\cos \alpha_1 - 2\phi] = 0$$

$$[2\cos \alpha_1 - 2\phi] = 0$$

$$\phi_{opt} = \cos \alpha_1$$

When  $\phi = \cos \alpha_1$ , the blade efficiency is the maximum, therefore

$$\eta_{b,max} = \frac{2[2\cos^2 \alpha_1 - \cos^2 \alpha_1]}{[1 + 2\cos^2 \alpha_1 - \cos^2 \alpha_1]}$$

$$\eta_{b,max} = \frac{2\cos^2 \alpha_1}{1 + \cos^2 \alpha_1}$$



## Chapter 5

### HYDRAULIC TURBINES

#### 5.1 Introduction:

The energy source which does not depend on thermal energy input to produce mechanical output is hydraulic energy. It may be either in the form of potential energy as we find in dams, reservoirs or in the form of kinetic energy in flowing water. Hydraulic turbines are the machines which convert the hydraulic energy in to mechanical energy.

#### 5.2 Classification of Hydraulic Turbines:

*Question No 5.1(a): Classify hydraulic turbines with examples. (VTU, Dec-06/Jan-07, May/Jun-10)*

**Answer:** Hydraulic turbines are classified based on the following important factors:

**1. Based on the action of water on blades** or the energy available at the turbine inlet, hydraulic turbines are classified as impulse and reaction turbines.

**Impulse turbine:** In this type of turbine the energy of the fluid entering the rotor is in the form of kinetic energy of jets. Example: Pelton turbine.

**Reaction turbine:** In this turbine the energy of the fluid entering the rotor is in the form of kinetic energy of jets and pressure energy of turbine. Example: Francis turbine and Kaplan turbine.

**2. Based on the direction of fluid flow through the runner,** turbine are classified as tangential flow turbine, radial flow turbine, axial flow turbine and mixed flow turbine.

**Tangential flow turbine:** In this type of turbine water strikes the runner along the tangential direction, these turbines are also known as peripheral flow turbines. Example: Pelton turbine.

**Radial flow turbine:** In this type of turbine water flow through the runner along the radial direction. Example: Francis turbine.

**Axial flow turbine:** In this type of turbine water flow through the runner along the axial direction. Example: Kaplan turbine.

**Mixed flow turbine:** In this type of turbine water enters the runner radially and leaves the runner axially. Example: Francis turbine.

**3. Based on specific speed of runner,** turbines are classified as low specific speed turbines, medium specific speed turbines and high specific speed turbines.

**Low specific speed turbines:** Such turbines have usually high head in the range of 200 m to 1700 m and these machines require low discharge. These turbines have specific speed in the range of 10 to 30 for single jet and 30 to 50 for double jet. Example: Pelton turbine

**Medium specific speed turbines:** Such turbines have usually medium head in the range of 50 m to 200 m and these machines require medium discharge. These turbines have specific speed in the range of 60 to 400. Example: Francis turbine.

**High specific speed turbines:** Such turbines have usually very low head in the range of 2.5 m to 50 m and these machines require high discharge. These turbines have specific speed in the range of 300 to 1000. Example: Kaplan turbine.

**Question No 5.1(b):** Mention the general characteristics features of Pelton, Francis and Kaplan turbines. (VTU, Dec-12)

**Answer:** Pelton wheel turbine is an impulse turbine. These turbines have usually high head in the range of 200 m to 1700 m and these machines require low discharge, hence the specific speed is low in the range of 10 to 30. In this type of turbine water strikes the runner along the tangential direction, these turbines are also known as peripheral (tangential) flow turbines.

Francis turbine is a reaction turbine. These turbines have usually medium head in the range of 50 m to 200 m and these machines require medium discharge, hence the specific speed is medium in the range of 60 to 400. In this type of turbine water enters radially and leaves axially or vice versa, these turbines are also known as mixed flow turbines.

Kaplan turbine is also a reaction turbine. These turbines have usually very low head in the range of 2.5 m to 50 m and these machines require high discharge, hence the specific speed is high in the range of 300 to 1000. In this type of turbine water flow through the runner along the axial direction, these turbines are also known as axial flow turbines.

### 5.3 Heads and Efficiencies of Hydraulic Turbines:

**Question No 5.2:** Define the following terms (i) Gross head (ii) Net head (iii) Volumetric efficiency (iv) Hydraulic efficiency (v) Mechanical efficiency (vi) Overall efficiency.

(VTU, Jul/Aug-04, Dec-07/Jan-08, Dec-12)

**Answer: (i) Gross head ( $H_g$ ):** It is the head of water available for doing useful work. It is the difference between the head race and tail race level when there is no flow. It is also known as static head.

**(ii) Net head ( $H$ ):** It is the head available at the inlet of the turbine. It is obtained by considering all losses, like loss in kinetic energy of water due to friction, pipe bends and fittings. If  $h_f$  is the total loss, then net head is given by  $H = H_g - h_f$ .

**(iii) Volumetric efficiency ( $\eta_v$ ):** It is the ratio of the quantity of water striking the runner of the turbine to the quantity of water supplied at the turbine inlet.

$$\eta_v = \frac{Q - \Delta Q}{Q}$$

Where  $\Delta Q$  is the amount of water that slips directly to the tail race.

(iv) **Hydraulic efficiency ( $\eta_H$ ):** It is the ratio of work done by the runner to the energy available at the inlet of the turbine.

$$\eta_H = \frac{U(V_{u1} \pm V_{u2})}{gH} = \frac{g(H - h_L)}{gH} = \frac{H - h_L}{H}$$

Where  $H$  is net head and  $h_L = (h_{Lr} + h_{Lc})$  is head loss in the runner and casing.

If leakage losses are considered then actual hydraulic efficiency is,

$$\eta_{H,act} = \frac{(Q - \Delta Q)U(V_{u1} \pm V_{u2})}{gQH} = \frac{g(Q - \Delta Q)(H - h_L)}{gQH} = \left(\frac{Q - \Delta Q}{Q}\right) \left(\frac{H - h_L}{H}\right)$$

Or,  $\eta_{H,act} = \eta_v \eta_H$

(v) **Mechanical efficiency ( $\eta_m$ ):** It is the ratio of shaft work output by the turbine to the work done by the runner.

$$\eta_m = \frac{W_{sft}}{U(V_{u1} \pm V_{u2})} = \frac{W_{sft}}{g(H - h_L)}$$

(vi) **Overall efficiency ( $\eta_o$ ):** It is the ratio of shaft work output by the turbine to the energy available at the inlet of the turbine.

$$\eta_o = \frac{W_{sft}}{gH}$$

$$\eta_o = \eta_H \eta_v \eta_m = \eta_{H,act} \eta_m$$

## 5.4 Pelton Wheel Turbines: *Keerthi Kumar N*

**Question No 5.3:** With a neat sketch explain the working of a Pelton turbine. What is the reason for the provision of a splitter in a Pelton wheel bucket?

**Answer:** Pelton wheel turbine is an impulse turbine working under high head and low discharge. In this turbine water carried from the penstock enters the nozzle emerging out in the form of high velocity water jet. The potential energy of water in the penstock is converted in to kinetic energy by nozzle which is used to run the turbine runner.

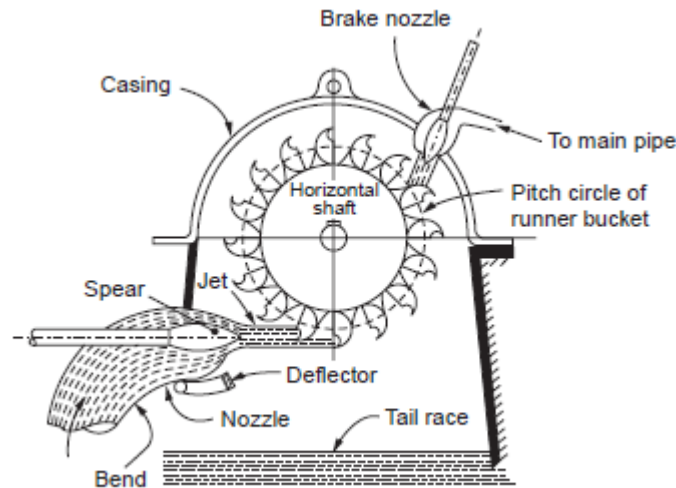


Fig. 5.1 Pelton wheel

Figure 5.1 shows main components of Pelton wheel, water from a high head source or reservoir like dam enters the turbine runner through large diameter pipes known as penstocks. Each penstock pipe is branched in such a way that it can accommodate a nozzle at the end. Water flows through these nozzles as a high speed jet striking the vanes or buckets attached to the periphery of the runner. The runner rotates and supplies mechanical work to the shaft. Water is discharged at the tail race after doing work on the runner.

In a Pelton wheel the jet of water strikes the bucket and gets deflected by the splitter into two parts, this negates the axial thrust on the shaft.

### 5.5 Force, Power and Efficiency of a Pelton Wheel:

**Question No 5.4:** Derive an expression for force, power and efficiency of a Pelton wheel with the help of velocity triangles. (VTU, Jun/Jul-14) Or, Obtain an expression for the workdone per second by water on the runner a Pelton wheel and hydraulic efficiency. (VTU, Jun/Jul-14)

**Answer:** The inlet and outlet velocity triangles for Pelton wheel turbine is as shown in figure 5.2. As the runner diameter is same at inlet and outlet, tangential velocity of wheel remains the same. In practical case the relative at the outlet is slightly less than that at inlet due to frictional loss over the inner surface of the bucket. Some velocity is also lost due to the jet striking over the splitter. Hence  $V_{r2} = C_b V_{r1}$ , where  $C_b$  is bucket velocity coefficient.

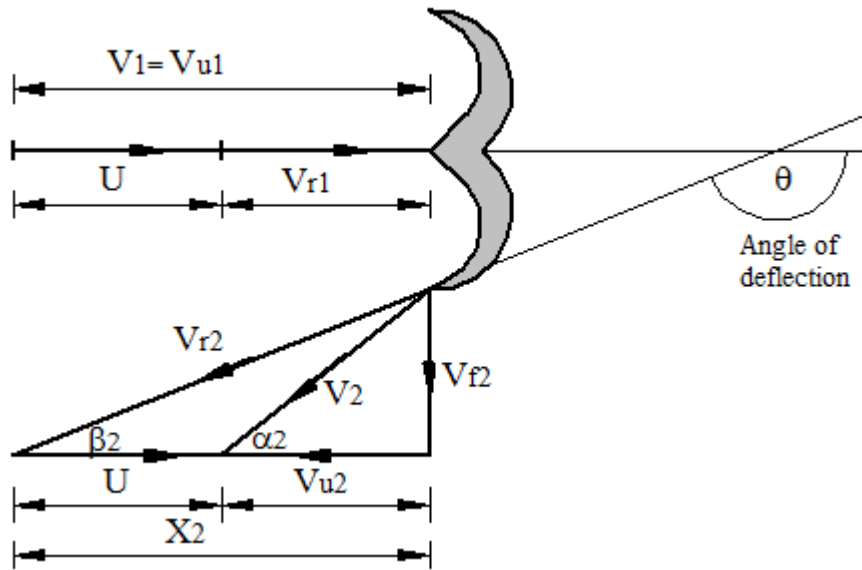
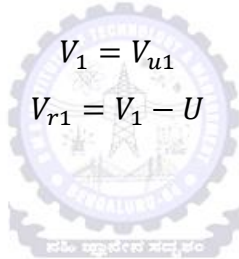


Fig. 5.2 Velocity triangles for Pelton wheel turbine

From inlet velocity triangle,



From outlet velocity triangle,

$$V_{r2} = C_b V_{r1}$$

$$V_{u2} = x_2 - U = V_{r2} \cos \beta_2 - U$$

$$V_{u2} = C_b V_{r1} \cos \beta_2 - U = C_b (V_1 - U) \cos \beta_2 - U$$

Then,  $(V_{u1} + V_{u2}) = V_1 + C_b (V_1 - U) \cos \beta_2 - U = (V_1 - U) + C_b (V_1 - U) \cos \beta_2$

$$(V_{u1} + V_{u2}) = (V_1 - U) [1 + C_b \cos \beta_2]$$

Force exerted by the jet on the wheel is,

$$F = \dot{m} (V_{u1} + V_{u2})$$

$$F = \rho Q (V_1 - U) [1 + C_b \cos \beta_2]$$

Power output by the wheel is,

$$P = FU$$

$$P = \rho Q U (V_1 - U) [1 + C_b \cos \beta_2]$$

Hydraulic efficiency of the wheel is,

$$\eta_H = \frac{U(V_{u1} + V_{u2})}{gH} = \frac{U(V_{u1} + V_{u2})}{\frac{V_1^2}{2}}$$

$$\eta_H = \frac{2U(V_1 - U)[1 + C_b \cos \beta_2]}{V_1^2}$$

Or,

$$\eta_H = \frac{2(UV_1 - U^2)}{V_1^2} [1 + C_b \cos \beta_2]$$

$$\eta_H = 2(\varphi - \varphi^2)[1 + C_b \cos \beta_2]$$

Where  $\varphi = \frac{U}{V_1}$  speed ratio.

**Question No 5.5:** Derive an expression for force, power and efficiency of a Pelton wheel assuming no frictional losses with the help of velocity triangles. (VTU, Dec-07/Jan-08)

**Answer:** By assuming no frictional losses over the blades the relative velocity of jet is remains constant, i.e.,  $V_{r2} = V_{r1}$

From inlet velocity triangle,

$$V_1 = V_{u1}$$

$$V_{r1} = V_1 - U$$

$$V_{r2} = V_{r1}$$

From outlet velocity triangle,

$$V_{u2} = x_2 - U = V_{r2} \cos \beta_2 - U$$

$$V_{u2} = V_{r1} \cos \beta_2 - U = (V_1 - U) \cos \beta_2 - U$$

Then,  $(V_{u1} + V_{u2}) = V_1 + (V_1 - U) \cos \beta_2 - U = (V_1 - U) + (V_1 - U) \cos \beta_2$

$$(V_{u1} + V_{u2}) = (V_1 - U)[1 + \cos \beta_2]$$

Force exerted by the jet on the wheel is,

$$F = \dot{m}(V_{u1} + V_{u2})$$

$$F = \rho Q(V_1 - U)[1 + \cos \beta_2]$$

Power output by the wheel is,

$$P = FU$$

$$P = \rho QU(V_1 - U)[1 + \cos \beta_2]$$

Hydraulic efficiency of the wheel is,

$$\eta_H = \frac{U(V_{u1} + V_{u2})}{gH} = \frac{U(V_{u1} + V_{u2})}{\frac{V_1^2}{2}}$$

$$\eta_H = \frac{2U(V_1 - U)[1 + \cos \beta_2]}{V_1^2}$$

Or,

$$\eta_H = \frac{2(UV_1 - U^2)}{V_1^2} [1 + \cos\beta_2]$$

$$\eta_H = 2(\varphi - \varphi^2)[1 + \cos\beta_2]$$

Where  $\varphi = \frac{U}{V_1}$  speed ratio

**Question No 5.6:** Show that for maximum utilization (maximum efficiency), the speed of the wheel is equal to half the speed of jet. (VTU, Dec-11)

**Answer:** The hydraulic efficiency of the Pelton wheel is given by,

$$\eta_H = 2(\varphi - \varphi^2)[1 + C_b \cos\beta_2]$$

For given machine  $C_b$  and  $\beta_2$  are constant, so  $\eta_H$  varies only with  $\varphi$ . The slope for the maximum hydraulic efficiency is,

$$\frac{d\eta_H}{d\varphi} = 0$$

$$\frac{d}{d\varphi} \{2(\varphi - \varphi^2)[1 + C_b \cos\beta_2]\} = 0$$

$$2(1 - 2\varphi)[1 + C_b \cos\beta_2] = 0$$

$$\varphi_{opt} = \frac{1}{2}$$

But  $\varphi = \frac{U}{V_1}$  speed ratio

$$\frac{U}{V_1} = \frac{1}{2}$$

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For maximum utilization (maximum hydraulic efficiency), the speed of the wheel is equal to half the speed of jet.

**Question No 5.7:** Draw the inlet and outlet velocity triangles for a Pelton wheel. Derive an expression for the maximum hydraulic efficiency of a Pelton wheel in terms of bucket velocity co-efficient and discharge blade angle. (VTU, Jun/Jul-11, Dec-14/Jan-15) Or,

Draw the inlet and outlet velocity triangles for a Pelton wheel. Show that the maximum hydraulic efficiency of a Pelton wheel turbine is given by,  $\eta_{H,max} = \frac{1+C_b \cos\beta_2}{2}$  where  $C_b$  is bucket velocity coefficient and  $\beta_2$  is exit blade angle. (VTU, Jun-12) Or,

Draw the inlet and outlet velocity triangles for a Pelton wheel. Show that the maximum hydraulic efficiency of a Pelton wheel turbine is given by,  $\eta_{H,max} = \frac{1+\cos\beta_2}{2}$ . Assume that relative velocity remains constant. (VTU, Dec-13)

**Answer:** The hydraulic efficiency of the Pelton wheel is given by,

$$\eta_H = 2(\varphi - \varphi^2)[1 + C_b \cos \beta_2]$$

When  $\varphi_{opt} = \frac{1}{2}$ , the hydraulic efficiency of Pelton wheel will be maximum.

$$\eta_{H,max} = 2 \left[ \frac{1}{2} - \left( \frac{1}{2} \right)^2 \right] [1 + C_b \cos \beta_2] = 2 \left[ \frac{1}{2} - \frac{1}{4} \right] [1 + C_b \cos \beta_2]$$

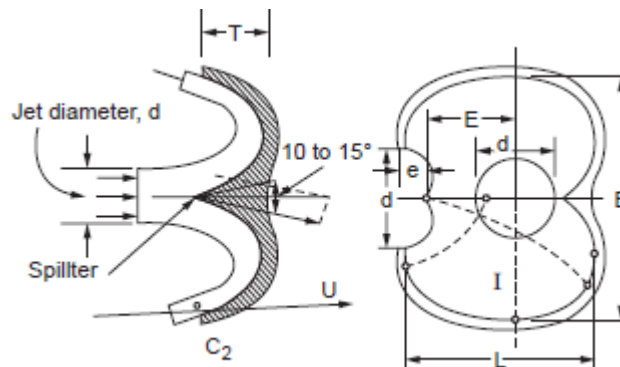
$$\eta_{H,max} = 2 \left[ \frac{1}{4} \right] [1 + C_b \cos \beta_2]$$

$$\eta_{H,max} = \frac{[1 + C_b \cos \beta_2]}{2}$$

If relative velocity remains constant (i.e. no frictional losses over the bucket),  $C_b = 1$

$$\eta_{H,max} = \frac{[1 + \cos \beta_2]}{2}$$

### 5.6 Design Parameters of Pelton Wheel:



1. Velocity of jet from the nozzle  $V_1 = C_v \sqrt{2gH}$

Where  $C_v$  is coefficient of velocity for nozzle ranges from 0.97 to 0.99

2. Tangential velocity of buckets  $U = \varphi \sqrt{2gH}$

Where  $\varphi$  is speed ratio varies ranges from 0.43 to 0.48

3. Least diameter of the jet (d):

$$\text{Total discharge, } Q = n \frac{\pi}{4} d^2 V_1$$

Where 'n' is number of jets or nozzles

4. Mean diameter or pitch diameter of buckets (D):

$$\text{Tangential velocity, } U = \frac{\pi D N}{60}$$

5. Jet ratio: It is the ratio of mean diameter of the runner to the minimum diameter of the jet.

$$J_r = \frac{D}{d} \text{ ranges between 6 to 35}$$

6. Minimum number of buckets:

$$Z = \frac{J_r}{2} + 15$$



7. Angle of deflection usually ranges from  $165^\circ$  to  $170^\circ$ , hence vane angle at outlet

$$\beta_2 = (180^\circ - \text{Angle of deflection})$$

8. Head loss due to friction in penstock:

$$h_f = \frac{4fLV_p^2}{2gD_p}$$

Where  $D_p$  is diameter of penstock,  $L$  is length of the penstock,  $V_p$  is fluid velocity through penstock and  $f$  is friction coefficient for penstock.

9. Bucket dimensions:

Width of the bucket  $B = 2.8d$  to  $4d$

Length of the bucket  $L = 2.4d$  to  $2.8d$

Depth of bucket  $T = 0.6d$  to  $0.95d$

10. Number of jets: Theoretically six jets or nozzles can be used with one Pelton wheel. However, practically not more than two jets per runner are used for a vertical runner and not more than four jets per runner are used for a horizontal runner.

### 5.7 Francis Turbine:

**Question No 5.8:** *With a neat sketch explain the working of Francis turbine. Draw the velocity triangles of Francis turbine. (VTU, Jul/Aug-05, Dec-06/Jan-07, Dec-09/Jan-10)*

**Answer:** Francis turbine is a reaction type turbine. Earlier Francis turbines were purely radial flow type but modern Francis turbines are mixed flow type in which water enters the runner radially and leaves axially at the centre. Figure 5.3 shows the main components of Francis turbines.

**(i) Scroll (spiral) casing:** It is also known as spiral casing. The water from penstock enters the scroll casing which completely surrounds the runner. The main function of spiral casing is to provide a uniform distribution of water around the runner and hence to provide constant velocity. In order to provide constant velocity, the cross sectional area of the casing gradually decreases as the water reaching the runner.

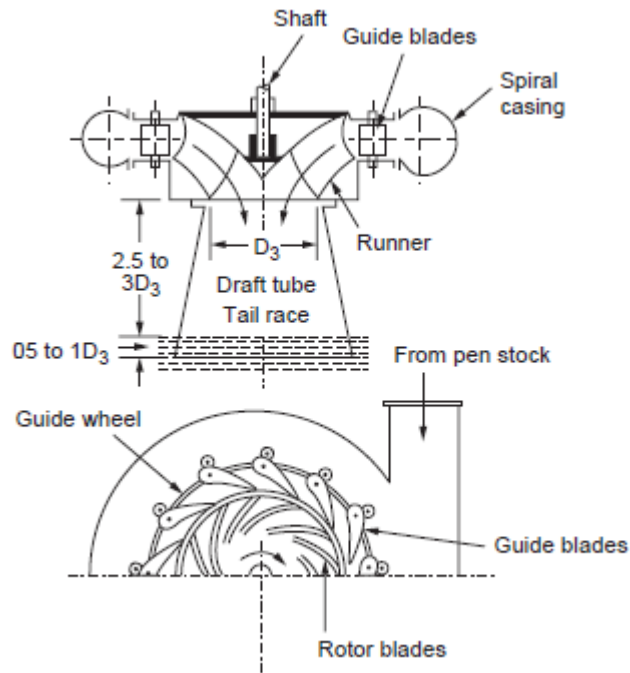


Fig. 5.3 Francis turbine

**(ii) Guide vanes (blades):** After the scroll ring water passes over to the series of guide vanes or fixed vanes, which surrounds completely around the turbine runner. Guide vanes regulate the quantity of water entering the runner and direct the water on to the runner.

**(iii) Runner (Rotor):** The runner of turbine is consists of series of curved blades evenly arranged around the circumference. The vanes or blades are so shaped that water enters the runner radially at outer periphery and leaves it axially at its centre. The change in direction of flow from radial to axial when passes over the runner causes the appreciable change in circumferential force which is responsible to develop power.

**(iv) Draft tube:** The water from the runner flows to the tail race through the draft tube. A draft tube is a pipe or passage of gradually increasing area which connect the exit of the runner to the tail race. The exit end of the draft tube is always submerged below the level of water in the tail race and must be airtight.

**Velocity triangles for Francis turbine:** In the slow, medium and fast runners of a Francis turbine the inlet blade angle ( $\beta_1$ ) is less than, equal to and greater than  $90^\circ$  respectively. The whirl component of velocity at the outlet is zero (i.e.,  $V_{u2}=0$ ).

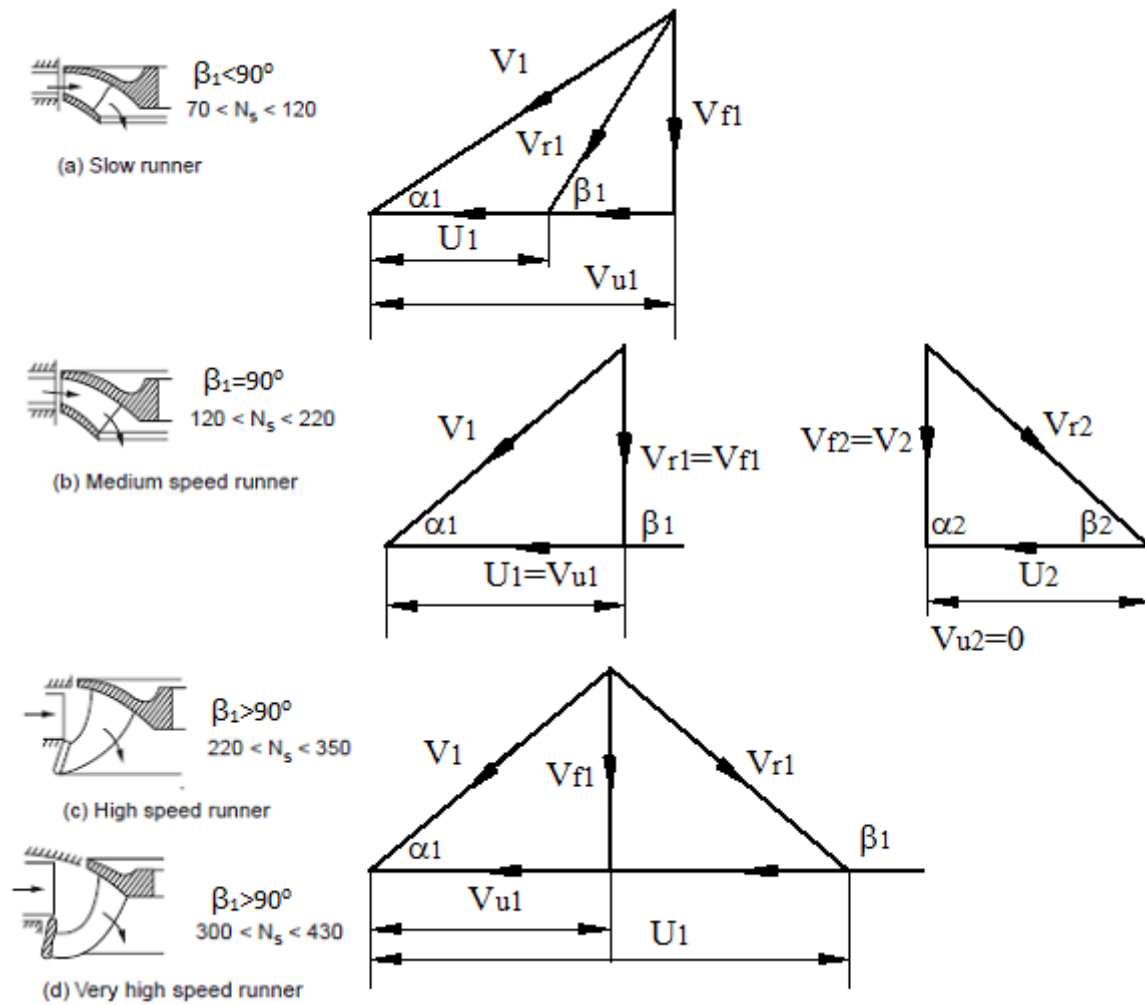


Fig. 5.4 Velocity triangle for Francis turbine

**5.8 Design Parameters of Francis Turbine:**

1. Flow velocity or radial velocity at the turbine inlet is given by,  $V_{f1} = \psi \sqrt{2gH}$

Where  $\psi$  is flow ratio ranging from 0.15 to 0.30

2. Tangential velocity of the runner or wheel at the inlet is given by,  $U_1 = \phi \sqrt{2gH}$

Where  $\phi$  is speed ratio ranging from 0.6 to 0.9

3. Diameter of runner:

Inlet diameter ( $D_1$ ) of the runner,  $U_1 = \pi D_1 N$

Outlet diameter ( $D_2$ ) of the runner,  $U_2 = \pi D_2 N$

Where  $U_1$  and  $U_2$  are inlet and outlet runner velocity respectively

4. Discharge at the outlet is radial then the guide blade angle at the outlet is  $90^\circ$ .

i.e.,  $\alpha_2 = 90^\circ$  and  $V_{u2} = 0$

5. Head at the turbine inlet assuming no energy loss is given by,  $gH = (U_1 V_{u1} \pm U_2 V_{u2}) + \frac{V_2^2}{2}$

$$H = \frac{1}{g} \left[ (U_1 V_{u1} \pm U_2 V_{u2}) + \frac{V_2^2}{2} \right]$$

6. Discharge through the turbine is given by,  $Q = A_f V_f = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$

Where  $A_f$  is area of flow through the runner,  $D$  is diameter of the runner,  $B$  is width of the runner and  $V_f$  is flow velocity.

If 'n' is the number of vanes in the runner and 't' is the thickness of the vane, then

$$Q = (\pi D_1 - nt_1) B_1 V_{f1} = (\pi D_2 - nt_2) B_2 V_{f2}$$

Normally it is assumed that,  $D_1 = 2D_2$ ,  $V_{f1} = V_{f2}$ , and  $B_2 = 2B_1$

7. Ratio of width to diameter is given by,  $r = \frac{B_1}{D_1}$  ranging from 0.10 to 0.38.

### 5.9 Kaplan Turbine:

**Question No 5.9:** Explain the functioning of a Kaplan turbine with the help of a sectional arrangement diagram. Draw the velocity triangles of Kaplan turbine. (VTU, Jul-07, Jun/Jul-08)

**Answer:** The Kaplan turbine is an axial flow reaction turbine in which the flow is parallel to the axis of the shaft as shown in figure 5.5. In which water enters from penstock into the spiral casing. The guide vanes direct water towards the runner vanes without shock or formation of eddies. Between the guide vanes and the runner, the fluid gets deflected by  $90^\circ$  so that flow is parallel to the axis of rotation of the runner which is known as axial flow. The guide vanes impart whirl component to flow and runner vanes nullify this effect making flow purely axial. As compared to Francis turbine runner blades (16 to 24 numbers) Kaplan turbine uses only 3 to 8 blades. Due to this, the contact surface with water is less which reduces frictional resistance and losses.

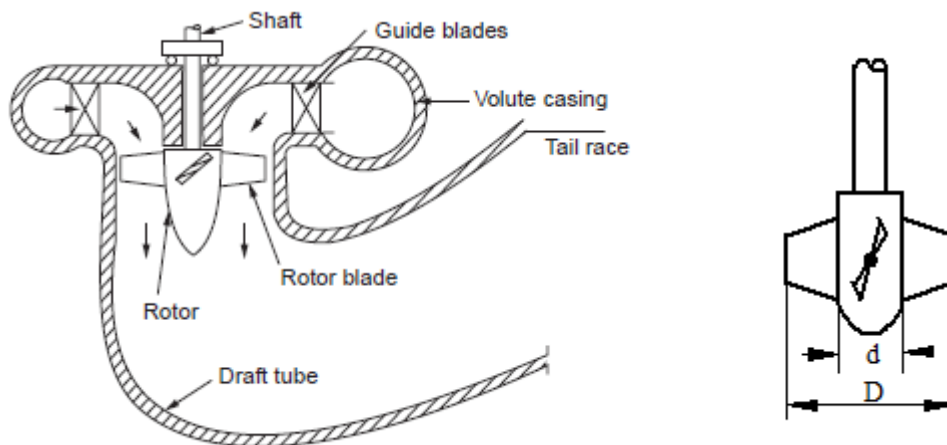


Fig. 5.5 Kaplan turbine

**Velocity triangles for Kaplan turbine:** At the outlet, the discharge is always axial with no whirl velocity component (i.e.,  $V_{u2}=0$ ). The inlet and outlet velocity triangles for Kaplan turbine are as shown in figure 5.6.

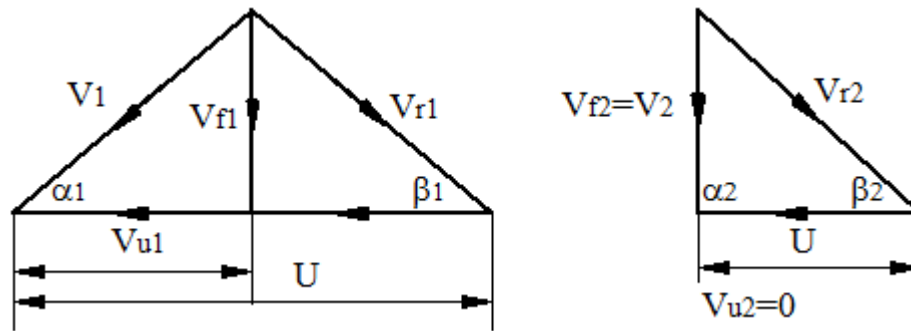


Fig. 5.6 Velocity triangles for Kaplan turbine

### 5.10 Design Parameters of Kaplan Turbine:

1. Flow velocity or radial velocity at the turbine inlet is given by,  $V_{f1} = \psi\sqrt{2gH}$

Where  $\psi$  is flow ratio ranging from 0.35 to 0.75

2. Flow velocity is remains constant throughout the runner,  $V_{f1} = V_{f2} = V_f$

3. Discharge through the runner is given by,

$$Q = \frac{\pi}{4} (D^2 - d^2) V_f$$

Where  $D$  is tip diameter or outer diameter of the runner and  $d$  is hub diameter or boss diameter of the runner.

4. Discharge at the outlet is axial then the guide blade angle at the outlet is  $90^\circ$ .

i.e.,  $\alpha_2 = 90^\circ$  and  $V_{u2} = 0$

5. Head at the turbine inlet assuming no energy loss is given by,

$$H = \frac{1}{g} \left[ U(V_{u1} \pm V_{u2}) + \frac{V_2^2}{2} \right]$$

### 5.11 Draft Tubes:

**Question No 5.10 (a):** Write a short note on draft tubes in a reaction hydraulic turbines.

(VTU, Dec-14/Jan-15)

Water, after passing through the runner is discharged through a gradually expanding tube called draft tube. The free end of the draft tube is submerged deep into the water. Thus the entire water passage from the head race to tail race is completely closed and hence doesn't come in contact with atmospheric air. It is a welded steel plate pipe or a concrete tunnel with gradually increasing cross sectional area at the outlet.

#### 5.11.1 Functions of Draft Tube:

**Question No 5.10 (b):** Explain the functions of a draft tube in a reaction hydraulic turbine.

(VTU, Jun/Jul-11, Dec-11, Dec-12)

**Answer:** The functions of a draft tube are as follows,

1. A reaction turbine is required to be installed above the tail race level for easy of maintenance work, hence some head is lost. The draft tube recovers this head by reducing the pressure head at the outlet to below the atmospheric level. It increases the working head of the turbine by an amount equal to the height of the runner outlet above the tail race. This creates a negative head or suction head.
2. Exit kinetic energy of water is a necessary loss in the case of turbine. A draft tube recovers part of this exit kinetic energy.
3. The turbine can be installed at the tail race level, above the tail race level or below the tail race level.

### 5.11.2 Types of Draft Tube:

**Question No 5.11:** Briefly explain with neat sketches different types of draft tube used in reaction hydraulic turbines. (VTU, Jun/Jul-09)

**Answer:** The important types of draft tubes are (i) Conical (straight divergent tube) type, (ii) Moody's bell mouthed tube type, (iii) Simple elbow type, and (iv) Elbow tube type having square outlet and circular inlet.

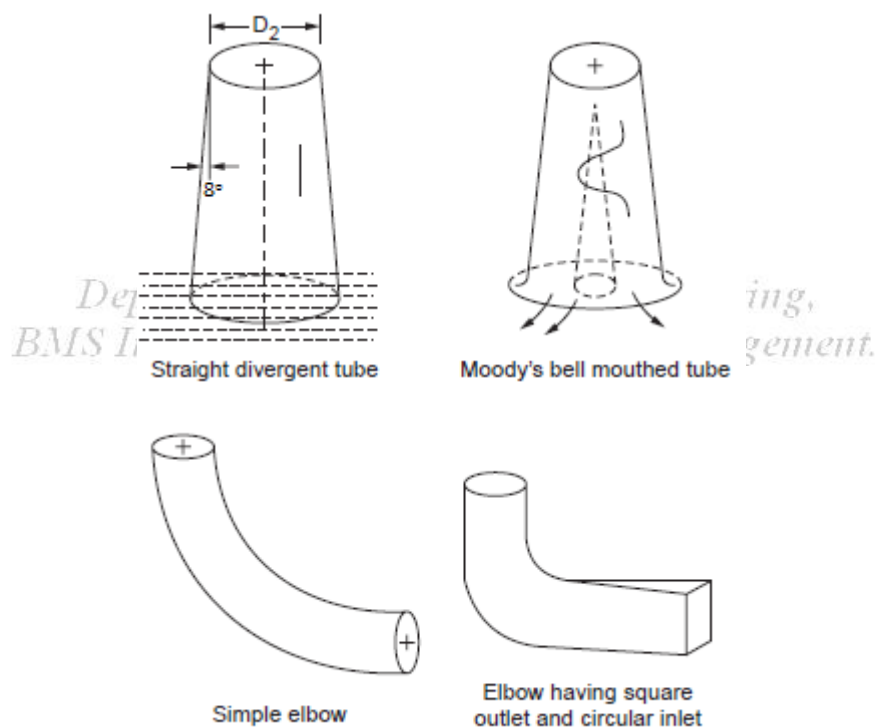


Fig. 5.7 Types of draft tube

The first form is the straight conical type stretching from the turbine to the tail-race. The second type is also a straight draft tube except that is bell-shaped. This type of draft tube has an advantage that it can allow flow with whirl component to occur with very small losses at the turbine exit.

The third form is the simple elbow tube, is used when the turbine must be located very close to or below the tail-race level. However, the efficiency of simple elbow tube is usually not as great as that of the first two types. The fourth form of draft tube is similar to the third one except that the exit shape is square or rectangular instead of cylindrical as in the simple elbow tube.

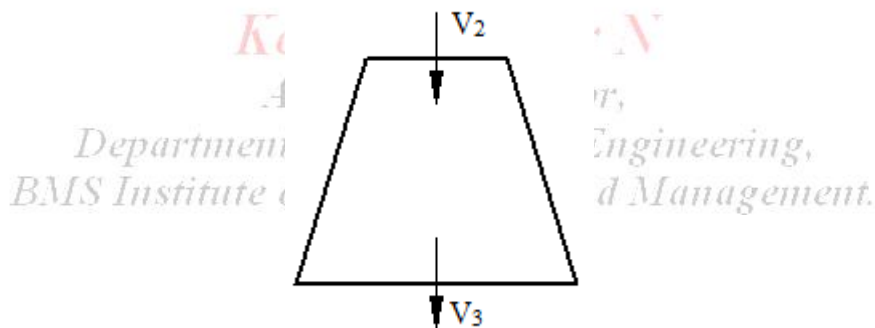
In these types, the conical type is most efficient and commonly used. For the straight divergent type conical draft tube, the centre cone angle should not exceed  $8^\circ$ . If this angle exceeds  $8^\circ$ , the water flowing through the draft tube will not remain in contact with its inner surface and hence eddies are formed and the efficiency will be reduced. Draft tube efficiencies range generally from 0.7 to 0.9 for the first two types while they are 0.6 to 0.85 for the elbow tube types.

### 5.11.3 Efficiency of Draft Tube:

**Question No 5.12:** Show that the efficiency of draft tube is given by  $\eta_d = \frac{V_2^2 - V_3^2 - 2gh_f}{V_2^2}$  where  $V_2$  is absolute velocity of water at rotor exit,  $V_3$  is absolute velocity of water at draft tube exit,  $h_f$  is loss of head due to friction. (VTU, Dec-08/Jan-09)

**Answer:** Efficiency of the draft tube is defined as the ratio of actual conversion of kinetic head into pressure head to the kinetic head available at the inlet of the draft tube.

Consider  $V_2$  is absolute velocity of water at rotor exit,  $V_3$  is absolute velocity of water at draft tube exit and  $h_f$  is loss of head due to friction.



Mathematically,

$$\eta_d = \frac{PH_{obt}}{KH_{avail}}$$

$$\eta_d = \frac{\frac{(V_2^2 - V_3^2)}{2g} - h_f}{\frac{V_2^2}{2g}}$$

$$\eta_d = \frac{V_2^2 - V_3^2 - 2gh_f}{V_2^2}$$

## Chapter 6

### CENTRIFUGAL PUMPS

#### 6.1 Introduction:

Centrifugal pump is a power absorbing turbomachine used to raise liquids from a lower level to a higher level by creating the required pressure with help of centrifugal action. Thus it can be defined as a machine which converts mechanical energy into pressure energy (hydraulic energy) by means of centrifugal action on the liquids.

When a certain amount of liquid is rotated by an external energy (mechanical energy) inside the pump casing, a forced vortex is set up, which raises the pressure head of the rotating liquid purely by centrifugal action.

#### 6.2 Working Principal:

Figure 6.1(a) shows the working principal of a centrifugal pump. The liquid to be pumped enters the centre of the impeller which is known as eye of the pump and discharge into the space around the casing and hence filling the space. Due to the rotation of the impeller inside the pump casing a forced vortex is set up which imparts pressure head to the liquid purely by centrifugal action.

The pressure head developed by centrifugal action is entirely by the velocity imparted to the liquid by rotating impeller and not due to any displacement or impact. Thus the mechanical action of the pump is to impart velocity to liquid so that its speed is enough to produce necessary centrifugal head for discharging.

#### 6.3 Classification of Centrifugal Pumps:

Centrifugal pumps are classified based on following aspects:

##### 6.3.1 Based on Type of Casings:

**Question No 6.1:** With neat sketch explain the various types of casings used in centrifugal pumps. (VTU, Jun/Jul-08, Jun/Jul-09)

**Answer:** Based on the type of casing centrifugal pumps may be classified as volute pump and diffusion pump.

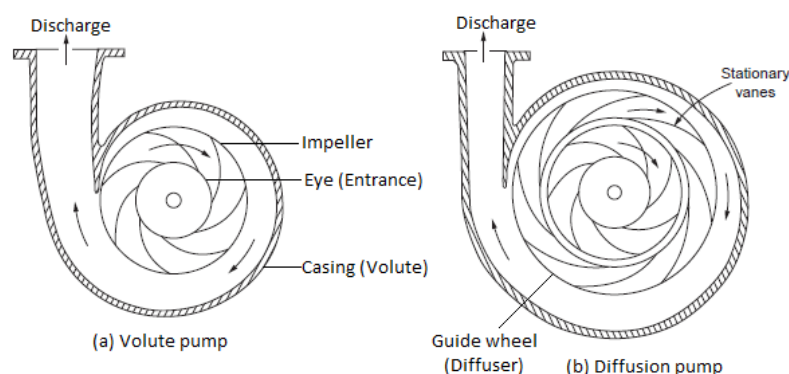




Fig. 6.1 Classification based on pump casings

**1. Volute Pump:** A volute pump is used to discharge water at a high velocity. The pump consists of a volute casing of spiral form with gradually increasing cross sectional area towards the discharge end. Mean velocity of flowing fluid remains constant as cross sectional area at any point is proportional to amount of water flowing through that section. Loss of kinetic head due to eddy formation is avoided due to the spiral shape. A volute pump is shown in figure 6.1(a).

The volute casing provided serves the following purpose:

- To collect water from the periphery of the impeller and to transmit it to the delivery pipe at a constant velocity.
- To eliminate the head loss due to change in velocity, as the velocity of water leaving the impeller equals the velocity of flow in the volute.
- To increase the efficiency of the pump by eliminating loss of head.

**2. Diffusion Pump or Turbine Pump:** The diffusion pump consists of an impeller surrounded by guide wheel fitted with stationary vanes or diffusers as shown in figure 6.1(b). The cross sectional area of these vanes increases gradually so that pressure of the fluid increases by decreasing velocity as fluid passes over the vanes. The guide vanes are so designed that the angle made at the entrance should perfectly match with the direction of absolute velocity at the outlet of the impeller.

Diffusion pumps may be either horizontal or vertical shaft type. The vertical shaft type pump is suitable for deep wells where space consideration is more important. These pumps are used in narrow wells and mines.

**6.3.2 Based on Fluid Entrance:** Based on fluid entrance centrifugal pumps are classified as single entry and double entry pumps as shown in figure 6.2.

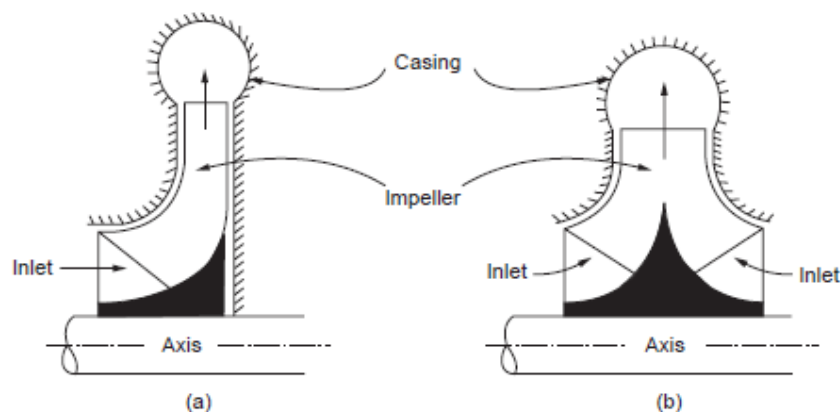


Fig. 6.2 Classification based on fluid entrance

**1. Single Entry Pump:** In single entry pumps water is admitted from one side of the impeller only.

**2. Double Entry Pump:** In double entry pumps water enters from both sides of the impeller. This arrangement neutralizes the axial thrust, producing high heads. It is used for pumping large quantities of liquid.

### 6.3.3 Based on Type of Impeller:

**Question No 6.2:** With a neat sketch explain different types of centrifugal pump impellers and list their merits and demerits.

**Answer:** Based on the type of impeller centrifugal pumps are classified as closed impeller pump, semi-closed (semi-open) impeller pump and open impeller pump as shown in figure 6.3.

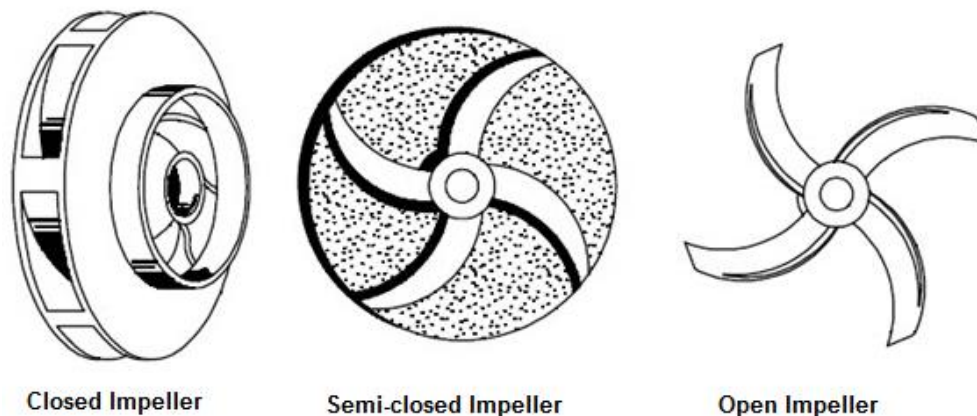


Fig. 6.3 Classification based on type of impeller

**1. Closed Impeller Pump:** If the impeller vanes are covered with shrouds on both sides of the impeller, then it is a closed impeller pump. This pump can handle non-viscous liquids like water (hot and cold), hot oil, acids etc.

**2. Semi-closed (Semi-open) Impeller Pump:** If the vanes are covered with shroud on one side of the impeller only, then it is a semi-closed impeller pump. The height of the vanes are increased and number of vanes are reduced so as avoid clogging of impeller. This pump can be used to discharge sewage water, pulp etc.

**3. Open Impeller Pump:** If the vanes are not covered with shroud, then it is open impeller pump. The impeller is made of forged steel and is designed to work under rough operating conditions. This pump can be used discharge mixtures of water, sand, clay etc.

The closed and semi-closed type impellers cover most industrial applications due to their superior performance and efficiency. The open type with no shroud is employed due to its simplicity, low cost and negligible maintenance. But its efficiency is very poor since it has poor flow confinement and the vanes are not formed to provide the best efficiency.

## 6.4 Heads of Centrifugal Pump:

**Question No 6.3:** Define the following with respect to centrifugal pumps: (i) Static head (ii) Manometric head (iii) Total head with the help of a schematic diagram. (VTU, Dec-09/Jan-10, May/June-10, Dec-12)

**Answer:** The different types of heads used in centrifugal pump are as shown in figure 6.4, and are defined as follows.

**1. Static Head ( $h$ ):** The main purpose of the pump is to lift water from the sump and deliver it at the overhead tank, the vertical height between the liquid level in the sump and the liquid level in the delivery tank is called the static head. It consists of two parts, the suction head ( $h_s$ ) and the delivery head ( $h_d$ ). i.e.,  $h = h_s + h_d$

Where suction head is the vertical height between the centre line of the pump to the liquid level in the sump and delivery head is the vertical height the centre line of the pump to the liquid level in the delivery tank.

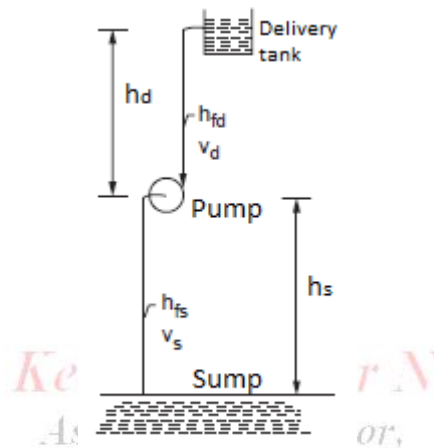


Fig. 6.4 Schematic diagram with suction and delivery pipes

**2. Manometric Head ( $H_m$ ):** It is the effective head that must be produced by the pump to satisfy the external requirements. It includes all the losses like frictional losses, leakage losses etc.

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_s^2}{2g}$$

$$H_m = h + h_{fs} + h_{fd} + \frac{V_s^2}{2g}$$

Where  $h_{fs}$  and  $h_{fd}$  are the head loss due to friction in suction and delivery pipes respectively and  $V_s$  is the velocity of the fluid in the suction pipe.

The head imparted to the liquid by the impeller is equal to sum of manometric head and loss of head ( $H_L$ ) in the impeller and casing.

$$\frac{U_2 V_{u2}}{g} = H_m + H_L$$

$$H_m = \frac{U_2 V_{u2}}{g} - H_L$$

The manometric head is measured by installing pressure gauges in the delivery and suction lines of the pump as close as possible to the pump inlet and the exit.

$$H_m = \frac{P_d - P_s}{\rho g}$$

Where  $P_s$  and  $P_d$  are the pressure denoted by the gauges on the suction and delivery sides respectively. The pressure at the suction of the pump is,

$$P_s = P_a - \rho g h_s - \rho g h_{fs} - \frac{\rho V_s^2}{2}$$

Similarly, pressure at the delivery of the pump is,

$$P_d = P_a + \rho g h_d + \rho g h_{fd}$$

Where  $P_a$  is the atmospheric pressure.

**3. Total Head ( $H_e$ ):** The total head is the net head produced by the pump to overcome the static head, the total loss in the system due to friction, turbulence, foot-valves and bends, etc., and to provide the kinetic energy of water at the delivery tank.

$$H_e = h + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

Where  $h_{fs}$  and  $h_{fd}$  are the head loss due to friction in suction and delivery pipes respectively and  $V_d$  is the velocity of the fluid in the delivery tank.

The manometric head and the total head are the same if the suction and delivery pipes have the same diameter (i.e.  $V_s = V_d$ ).

#### 4. Net Positive Suction Head (NPSH):

**Question No 6.4:** What do you mean by NPSH? Is it desirable to have a lower or higher value of NPSH? Justify your answer with the help of relevant equations.

**Answer:** It is the head required at the pump inlet to keep the local pressure everywhere inside the pump above the vapour pressure.

Net positive suction head (NPSH) is defined as the difference between the pump's suction stagnation pressure head and vapour pressure head.

$$NPSH = \left[ \frac{P_s}{\rho g} + \frac{V_s^2}{2g} \right] - \frac{P_v}{\rho g}$$

Where  $V_s$  is the velocity of the water in suction side,  $P_s$  and  $P_v$  are the static pressure at the suction and the vapour pressure respectively.

In the suction the fluid is at atmospheric temperature so the vapour pressure remains constant, to increase the static head one has to increase net positive suction head. Therefore NPSH should be

higher value. In order to have cavitation free operation of centrifugal pump available NPSH should be greater than the minimum NPSH.

$$h = \frac{P_s}{\rho g} = NPSH + \frac{P_v}{\rho g} - \frac{V_s^2}{2g}$$

## 6.5 Efficiencies of Centrifugal Pump:

**Question No 6.5:** Explain the following with mathematical expression: (i) Manometric efficiency (ii) Mechanical efficiency (iii) Volumetric efficiency (iv) Overall efficiency. (VTU, Jun-12)

**Answer:** The different types of efficiencies expressed in centrifugal pump are as follows:

**1. Manometric Efficiency:** It is the ratio of the manometric head to the ideal head imparted by the impeller to the fluid.

$$\eta_{ma} = \frac{H_m}{\left(\frac{U_2 V_{u2}}{g}\right)} = \frac{\left(\frac{U_2 V_{u2}}{g}\right) - \text{Hydraulic losses}}{\left(\frac{U_2 V_{u2}}{g}\right)} = \frac{H_m}{H_m + \text{Hydraulic losses}}$$

**2. Mechanical Efficiency:** It is the ratio of the energy transferred by the impeller to the fluid to the mechanical energy delivered to the pump at the shaft.

$$\eta_m = \frac{W_{impeller}}{W_{shaft}} = \frac{U_2 V_{u2}}{U_2 V_{u2} + \text{Mechanical losses}}$$

**3. Volumetric Efficiency:** It is the ratio of the amount of fluid delivered by the delivery pipe to the amount of fluid entering the impeller through suction pipe.

$$\eta_v = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q_d}{Q_s} = \frac{Q_d}{Q_d + Q_L}$$

Where  $Q_L$  is the amount of fluid leakage loss.

**4. Overall Efficiency:** It is the ratio of actual hydraulic energy output by the pump to the mechanical energy input to the pump at the shaft. The overall pump efficiency is the product of hydraulic efficiency, volumetric efficiency and mechanical efficiency.

$$\eta_o = \eta_H \eta_v \eta_m$$

Where hydraulic efficiency is the ratio of the useful pump output head (static head) to the ideal head imparted by the impeller to the fluid

$$\eta_H = \frac{h}{\left(\frac{U_2 V_{u2}}{g}\right)}$$

## 6.6 Cavitation and Priming:

**Question No 6.6:** What is cavitation in centrifugal pump? What are the causes of cavitation? Explain the steps to be taken to avoid cavitation. (VTU, Jun/Jul-11, Dec-11)

**Answer:** If the pressure at any point in a suction side of centrifugal pump falls below the vapour pressure, then the water starts boiling forming saturated vapour bubbles. Thus, formed bubbles moves at very high velocity to the more pressure side of the impeller blade and strikes the surface of the blade and collapse there. In this way, as the pressure further decreases, more bubbles will be formed and collapses on the surface of the blades, physically enables to erosion and pitting, forming a cavities on blades. This process takes place many thousand times in a second and damages the blade of a centrifugal pump. This phenomenon is known as cavitation.

*Causes of cavitation:* The causes of cavitation are as follows,

1. The metallic surfaces damaged and cavities are formed on the impeller surface.
2. Considerable noise and vibration are produced due to the sudden collapse of vapour bubble.
3. The efficiency of the machine reduces.

*Steps to avoid cavitation:* The following steps should be taken to avoid cavitation,

1. The suction losses should be minimised through the use of large diameter suction tubes with fewer bends than in the delivery pipe.
2. The pressure of the fluid flow in any part of the system should not fall below the vapour pressure.
3. The impeller should be made of better cavitation resistant materials such as aluminium, bronze and stainless steel.

**Question No 6.7:** *Explain the term priming related to centrifugal pump. Why priming is necessary for a centrifugal pump. (VTU, Dec-07/Jan-08) Or,*

*What is priming? How priming will be done in centrifugal pumps? (VTU, Dec-12)*

**Answer:** Priming is the process of removing the air present in the suction pipe and impeller casing. To remove the air the suction pipe, casing of the pump and portion of the delivery pipe are completely filled with water before starting the pump. If the pump is running with air it develops the head in meters of air. If the pump is running with water, the head produced is in terms of meters of water. But as the density of air is very low, the head generated in terms of equivalent of water is negligible. It is therefore the pressure rise by the pump, when the air is present, may not be sufficient to suck water from the sump. To eliminate this difficulty, the pump is to be primed with water properly before start.

### **6.7 Work Done by the Centrifugal Pump:**

**Question No 6.8:** *Derive a theoretical head capacity (H-Q) relationship for centrifugal pumps and compressors and explain the influence of outlet blade angle. (VTU, Jul/Aug-05, Dec-11)*

**Answer:** The velocity diagram for centrifugal pumps and compressor with  $V_{u1} = 0$  is as shown in figure 6.5.

Energy transfer of a centrifugal compressor and pump is given as:

$$e = gH = U_2V_{u2} - U_1V_{u1}$$

(Because, in centrifugal pump and compressor the working fluid is usually water or oil)

Or,  $gH = U_2V_{u2}$  (Because,  $V_{u1} = 0$ )

From outlet velocity triangle,  $V_{u2} = U_2 - x_2$

But,  $\cot\beta_2 = \frac{x_2}{V_{m2}} \Rightarrow x_2 = V_{m2}\cot\beta_2$

$$V_{u2} = U_2 - V_{m2}\cot\beta_2$$

Therefore,  $gH = U_2(U_2 - V_{m2}\cot\beta_2)$

Or,  $H = \frac{U_2^2}{g} - \frac{U_2V_{m2}}{g}\cot\beta_2$

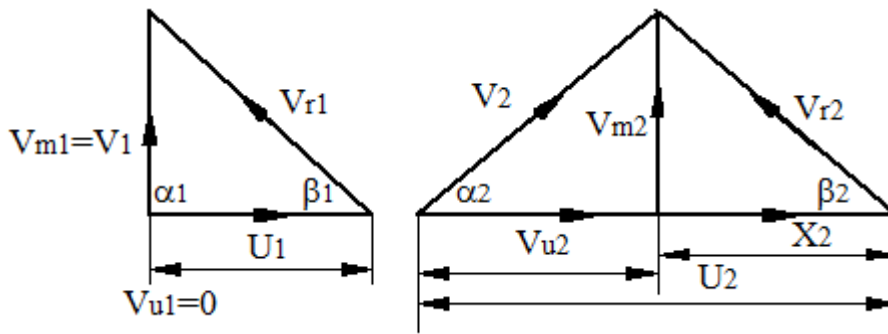


Fig. 6.5 Velocity triangles for centrifugal pump

Discharge at outer radius of centrifugal machine = Area of flow × Flow velocity

$$Q = \pi D_2 B_2 \times V_{m2}$$

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Department of Mechanical Engineering,  
BMS Institute of Technology and Management.

Then,

$$H = \frac{U_2^2}{g} - \left(\frac{U_2}{g}\right) \left(\frac{Q}{\pi D_2 B_2}\right) \cot\beta_2$$

By using above equation, H-Q characteristic curve of a given impeller exit blade angle  $\beta_2$  for different values of discharge is drawn in figure 6.6.

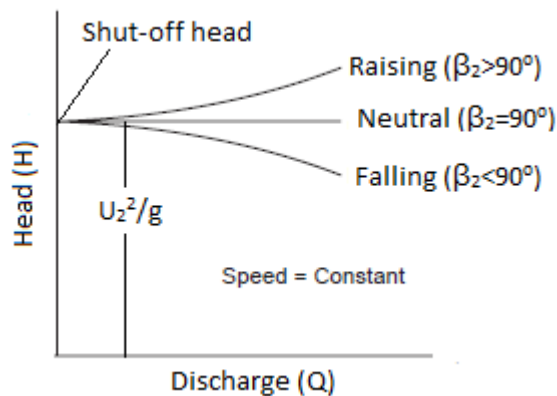


Fig. 6.6 H-Q characteristic curve

**Question No 6.9:** Draw the inlet and outlet velocity triangles for a radial flow power absorbing turbomachines with (i) Backward curved vane (ii) Radial vane (iii) Forward curved vane. Assume inlet whirl velocity to be zero. Draw and explain the head-capacity relations for the above 3 types of vanes. (VTU, Dec-08/Jan-09)

**Answer:** There are three types of vane shapes in centrifugal machines namely, (i) Backward curved vane (ii) Radial vane (iii) Forward curved vane.

The vane is said to be backward curved if the angle between the rotor blade-tip and the tangent to the rotor at the exit is acute ( $\beta_2 < 90^\circ$ ). If it is a right angle ( $\beta_2 = 90^\circ$ ) the blade said to be radial and if it is greater than  $90^\circ$ , the blade is said to be forward curved. Here the blade angles measured with respect to direction of rotor (clockwise direction). The velocity triangles at the outlet of centrifugal machines are shown figure 6.7.

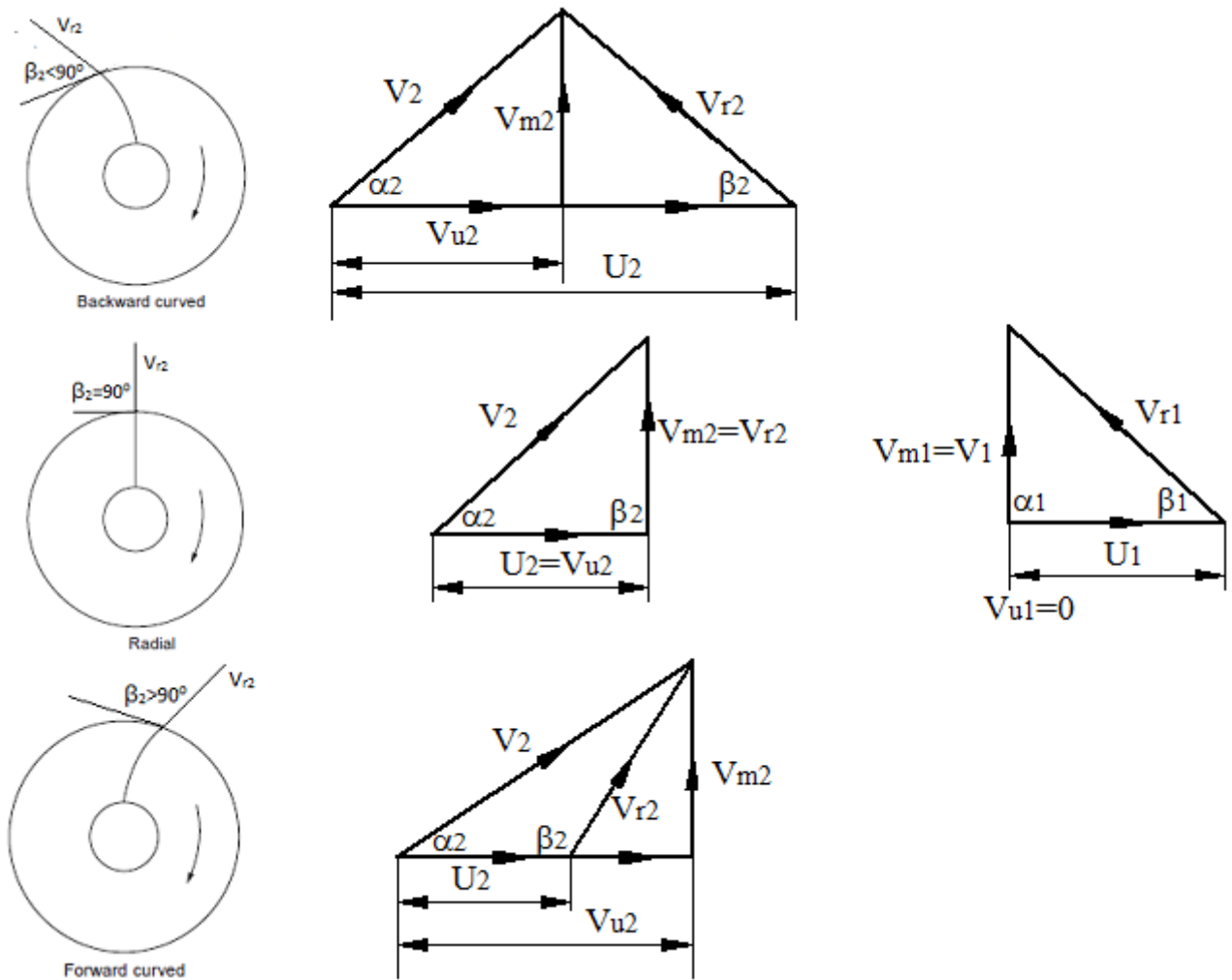


Fig. 6.7 Types of centrifugal vanes

The head-capacity characteristic curve for the above 3 types of vanes is given in figure 6.6, if  $\beta_2$  lies between  $0$  to  $90^\circ$  (backward curved vanes),  $\cot\beta_2$  in H-Q relation is always positive. So for backward curved vanes the head developed by the machine falls with increasing discharge. For values



of  $\beta_2$  between  $90^\circ$  to  $180^\circ$ ,  $\cot\beta_2$  in H-Q relation is negative. So for forward curved vanes the head developed by the machine continuously rise with increasing discharge. For  $\beta_2=90^\circ$  (radial vanes), the head is independent of flow rates and is remains constant. For centrifugal machines usually the absolute velocity at the entry has no tangential component (i.e.,  $V_{u1}=0$ ), thus the inlet velocity triangle for all the 3 types of vanes is same.

### 6.8 Static Pressure Rise in Centrifugal Pump:

**Question No 6.10:** Derive an expression for the static pressure rise in the impeller of a centrifugal pump with velocity triangles. (VTU, Dec-06/Jan-07, Jun/Jul-14) Or, For a centrifugal pump, show that the static head rise in the impeller neglecting the friction and other losses is given by  $\frac{1}{2g} [V_{m1}^2 + U_2^2 - V_{m2}^2 \text{cosec}^2 \beta_2]$  where  $V_{m1}$  and  $V_{m2}$  are velocities of flow at inlet and outlet,  $U_2$  is tangential velocity of impeller at outlet and  $\beta_2$  is vane angle at outlet.

(VTU, Jan/Feb-06, Dec-08/Jan-09, Dec-09/Jan-10)

**Answer:** The velocity diagram of a centrifugal pump is given by,

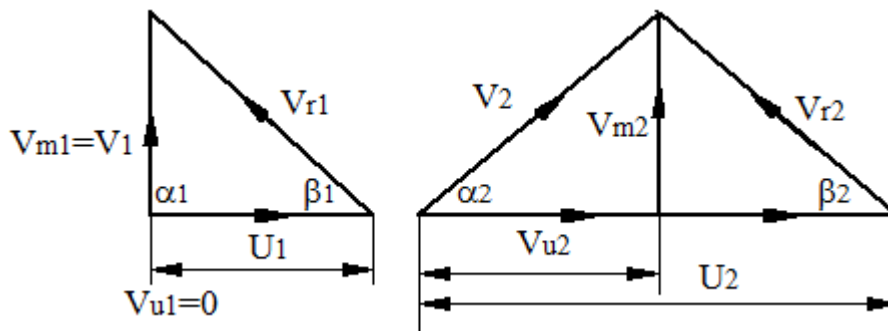


Fig. 6.8 Velocity triangles for centrifugal pump

Energy transfer due to static pressure change is given by,

$$e_{static} = \frac{(U_2^2 - U_1^2)}{2} - \frac{(V_{r2}^2 - V_{r1}^2)}{2}$$

From inlet velocity triangle,  $V_{r1}^2 = U_1^2 + V_{m1}^2$

From outlet velocity triangle,  $V_{r2}^2 = V_{m2}^2 + x_2^2$

But,  $\cot\beta_2 = \frac{x_2}{V_{m2}} \Rightarrow x_2 = V_{m2} \cot\beta_2$

Then,  $V_{r2}^2 = V_{m2}^2 + V_{m2}^2 \cot^2 \beta_2$

Then,

$$e_{static} = \frac{(U_2^2 - U_1^2)}{2} - \frac{(V_{m2}^2 + V_{m2}^2 \cot^2 \beta_2 - U_1^2 - V_{m1}^2)}{2}$$

$$e_{static} = \frac{1}{2} [V_{m1}^2 + U_2^2 - V_{m2}^2 (1 + \cot^2 \beta_2)]$$

Therefore static energy rise is given by,

$$e_{static} = \frac{1}{2} [V_{m1}^2 + U_2^2 - V_{m2}^2 \operatorname{cosec}^2 \beta_2]$$

The static pressure rise is given by,  $(P_2 - P_1) = \rho e_{static}$

$$(P_2 - P_1) = \frac{\rho}{2} [V_{m1}^2 + U_2^2 - V_{m2}^2 \operatorname{cosec}^2 \beta_2]$$

The static head rise is given by,  $h = \frac{(P_2 - P_1)}{\rho g} = \frac{e_{static}}{g}$

$$\frac{(P_2 - P_1)}{\rho g} = \frac{1}{2g} [V_{m1}^2 + U_2^2 - V_{m2}^2 \operatorname{cosec}^2 \beta_2]$$

## 6.9 Minimum Starting Speed:

**Question No 6.11:** What is minimum starting speed of a centrifugal pump? Obtain an expression for the minimum starting speed of a centrifugal pump.

(VTU, Jun/Jul-09, May/June-10, Jun/Jul-14, Dec-14/Jan-15)

**Answer:** When the pump is started there will be no flow until the pressure rise in the impeller is more than or equal to the manometric head. In other words the centrifugal head should be greater than the manometric head. Therefore, minimum starting speed is the speed of centrifugal pump at which centrifugal head is equal to manometric head.

For minimum starting speed condition, *centrifugal head* = *manometric head*

$$\frac{(U_2^2 - U_1^2)}{2g} = H_m = \frac{\eta_{ma} U_2 V_{u2}}{g}$$

$$\frac{\left[ \left( \frac{\pi D_2 N}{60} \right)^2 - \left( \frac{\pi D_1 N}{60} \right)^2 \right]}{2} = \eta_{ma} \left( \frac{\pi D_2 N}{60} \right) V_{u2}$$

$$\frac{\left( \frac{\pi N}{60} \right)^2 [D_2^2 - D_1^2]}{2} = \eta_{ma} \left( \frac{\pi N}{60} \right) D_2 V_{u2}$$

$$\frac{\pi N D_2^2 \left[ 1 - \frac{D_1^2}{D_2^2} \right]}{120} = \eta_{ma} D_2 V_{u2}$$

$$\frac{\pi N D_2 \left[ 1 - \frac{D_1^2}{D_2^2} \right]}{120} = \eta_{ma} V_{u2}$$

Then minimum starting speed in rpm is,

$$N_{min} = \frac{120 \eta_{ma} V_{u2}}{\pi D_2 \left[ 1 - \frac{D_1^2}{D_2^2} \right]}$$

## 6.10 Multistage Pump:

**Question No 6.12:** Write a note on multistage centrifugal pumps. (VTU, Dec-06/Jan-07)

**Question No 6.13:** What are the applications of multi-stage centrifugal pumps? With a neat sketch, explain centrifugal pumps in series and parallel. (VTU, Dec-12)

**Answer:** If the centrifugal pump consists of two or more impellers connected in series on the same shaft, then the pump is called multistage pump. The water enters the impeller 1 of a multistage pump with two impellers as shown in figure 6.9 and pressure increases in it. The high pressure water from impeller 1 is then entering the impeller 2 where the pressure increases further. The flow generally is same in both the impeller. The head produced by the combined impellers will be higher than either of one, but need not be sum of them. If 'n' be the number of identical impellers and each produce a head of  $H_m$ , then the total head produced is given by  $H_T = nH_m$ .

Multistage pump is used for the following reasons:

1. To decrease the size of the impeller.
2. To develop high head.
3. To keep the outlet blade angle less than  $45^\circ$ .

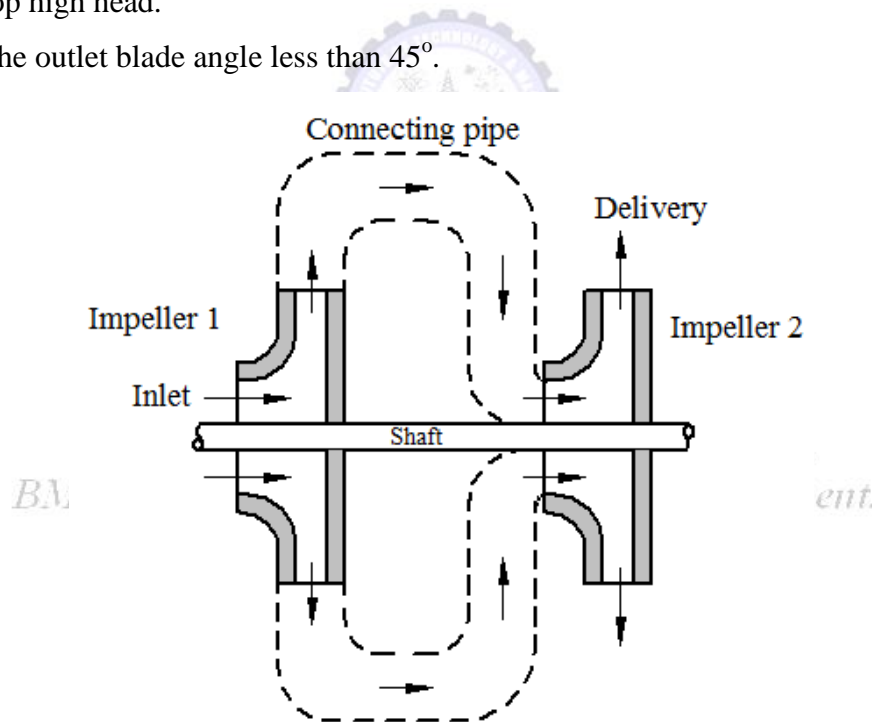


Fig. 6.9 Two stage centrifugal pump

### 6.11 Double-suction Pump:

In a double-suction pump two identically designed separate single stage pumps placed back-to-back (connected in parallel) as shown in figure 6.10. They draw water from a common source at a low level and discharge water to a common tank through a single delivery pipe. Each of the pump works against the same head. If 'n' be the number of identical impellers, each delivers the same flow rates which works under same head, then the total discharge is given by  $Q_T = nQ$ .

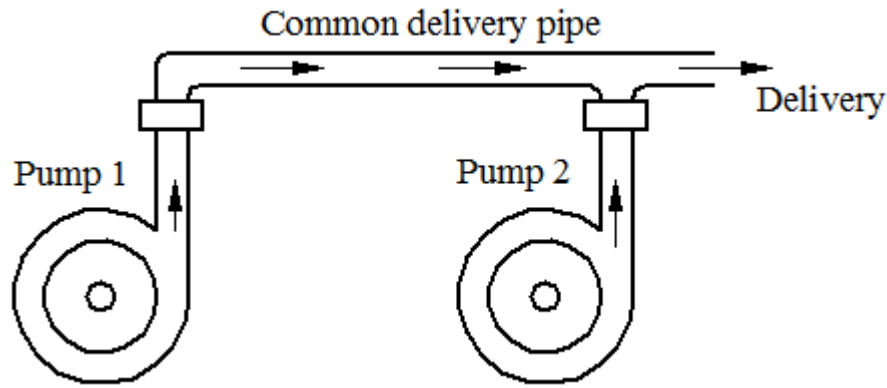


Fig. 6.10 Pumps connected in parallel

**6.12 Slip and Slip Co-efficient:**

In deriving the Euler’s equation, it was assumed that the velocity and pressure distributions are uniform over the impeller cross-sectional area. But in actual practice this assumption is not correct, because the velocity and pressure are not uniform over an impeller cross-sectional area as shown in figure 6.11. Due to uneven pressure distribution and hence the velocity distribution the tangential component of the velocity (whirl velocity) reduces, thus head developed by the machine is always less than that developed at the ideal condition.

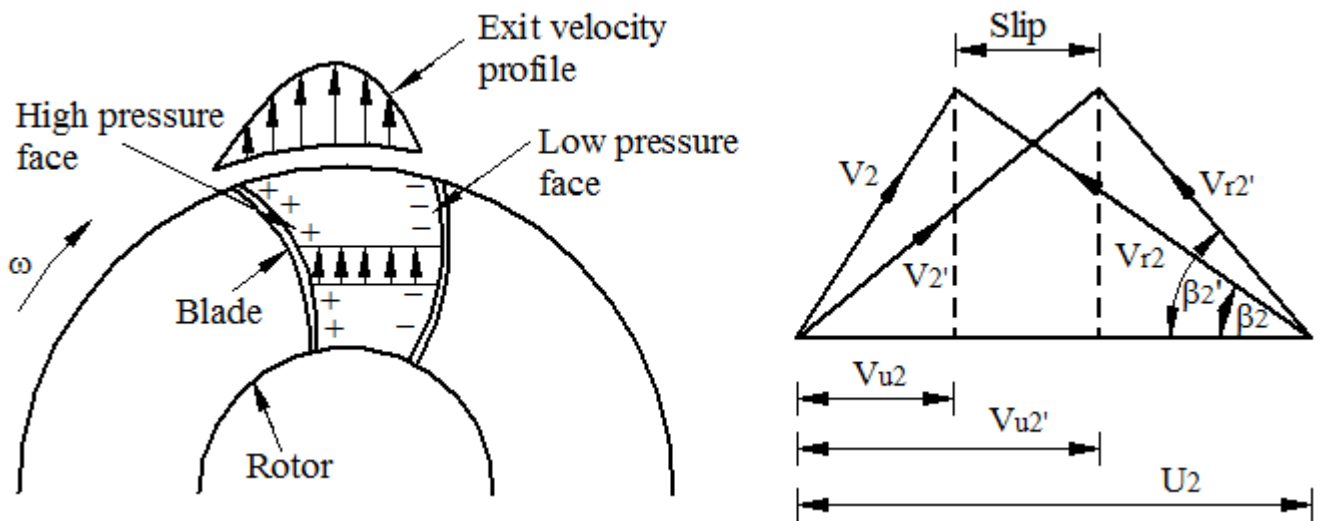


Fig. 6.11 Velocity and pressure distribution over impeller

Thus slip may be defined as the phenomenon observed in centrifugal machines due to uneven pressure distribution and the velocity distribution, which results in the reduction of tangential component of the velocity (whirl velocity).

If  $V_{u2'}$  is the tangential component of the velocity without slip and  $(V_{u2})$  is the tangential component of the velocity with slip, then slip ( $S$ ) is  $S = V_{u2'} - V_{u2}$

The ratio of ideal head ( $H_i$ ) with slip to the Euler’s head ( $H_e$ ) without slip is called the slip co-efficient ( $\mu$ ).

$$\mu = \frac{H_i}{H_e} = \frac{V_{u2}}{V_{u2'}}$$

Then work done on fluid by a centrifugal pump with slip is given as:

$$w = gH = \mu U_2 V_{u2'}$$

Although above equation modified by the slip coefficient, it is still the theoretical work done on the fluid, since slip will be present even if the fluid is frictionless (ideal fluid).



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## Chapter 7

# CENTRIFUGAL COMPRESSORS

### 7.1 Introduction:

**Question No 7.1:** Distinguish between fans, blowers and compressors and mention one application area for each. Or, Classify centrifugal compressors based on pressure developed. (VTU, Jul-07)

**Answer:** Centrifugal compressors work very much like centrifugal pumps except that they handle gases instead of liquids. Compressors as well as blowers and fans are the devices used to increase the pressure of a compressible fluid (gas).

A fan usually consists of a single rotor with or without a stator element and causes only a small rise in stagnation pressure of the flowing fluid, perhaps as low as 20 to 30 mm of water and very rarely in excess of 0.07 bar. Fans are used to provide strong circulating air currents or for air circulation and ventilation of buildings.

A blower may consist of one or more stages of compression with the rotors mounted on a common shaft. The air is compressed in a series of successive stages and is often led through a diffuser located near the exit. Blowers may run at very high shaft speeds and cause overall pressure rise in the range 1.5 to 2.5 bar. Blowers are used in ventilators, power stations, workshops, etc.

A compressor is a device used to produce large pressure changes ranging from 2.5 to 10 bar or more. Centrifugal compressors are mainly used in turbo-chargers.

The advantages of centrifugal compressor over the axial flow compressor are smaller length and wide range of mass flow rate of gas. The disadvantages are larger frontal area and lower maximum efficiency.

### 7.2 Components of Centrifugal Compressor:

**Question No 7.2:** Explain the various components of typical centrifugal compressors with the help of a schematic diagram. Discuss the actual pressure and velocity variations of flow across the impeller and diffuser. (VTU, Jul-07)

**Answer:** The principal components are the impeller and the diffuser. When the impeller is rotating at high speed, air is drawn in through the eye of the impeller. The absolute velocity of the inflow air is axial. The air then flows radially through the impeller passages due to centrifugal force. The total mechanical energy driving the compressor is transmitted to the fluid stream in the impeller where it is converted into kinetic energy, pressure and heat due to friction. The function of the diffuser is to convert the kinetic energy of air that leaving the impeller into pressure. The air leaving the diffuser is collected in a spiral casing from which it is discharged from compressor. The pressure and velocity variation across the centrifugal compressor is shown in figure 7.1.

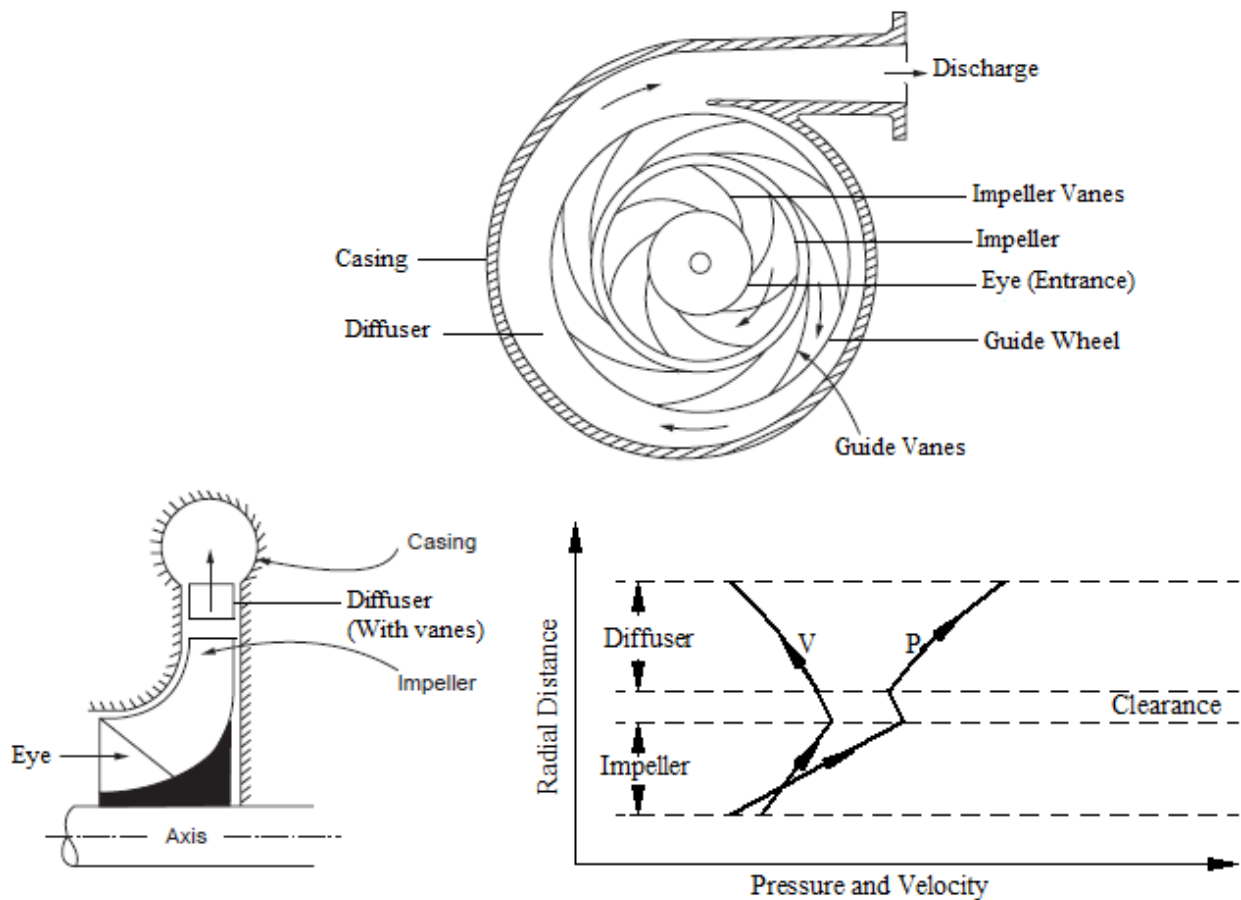


Fig. 7.1 Pressure and velocity diagram for centrifugal compressor

**7.3 Types of Vane Shapes:**

**Question No 7.3:** With a neat sketch and velocity triangles, explain different vane shapes of the centrifugal compressor. Draw the inlet velocity triangle assuming  $V_{u1} = 0$ . (VTU, May/Jun-10)

**Answer:** There are three types of vane shapes in centrifugal machines namely, (i) Backward curved vane (ii) Radial vane (iii) Forward curved vane.

The vane is said to be backward curved if the angle between the rotor blade-tip and the tangent to the rotor at the exit is acute ( $\beta_2 < 90^\circ$ ). If it is a right angle ( $\beta_2 = 90^\circ$ ) the blade said to be radial and if it is greater than  $90^\circ$ , the blade is said to be forward curved. Here the blade angles measured with respect to direction of rotor (clockwise direction). The velocity triangles at the outlet of centrifugal machines are shown below.

For centrifugal machines usually the absolute velocity at the entry has no tangential component (i.e.,  $V_{u1} = 0$ ), thus the inlet velocity triangle for all the 3 types of vanes is same.

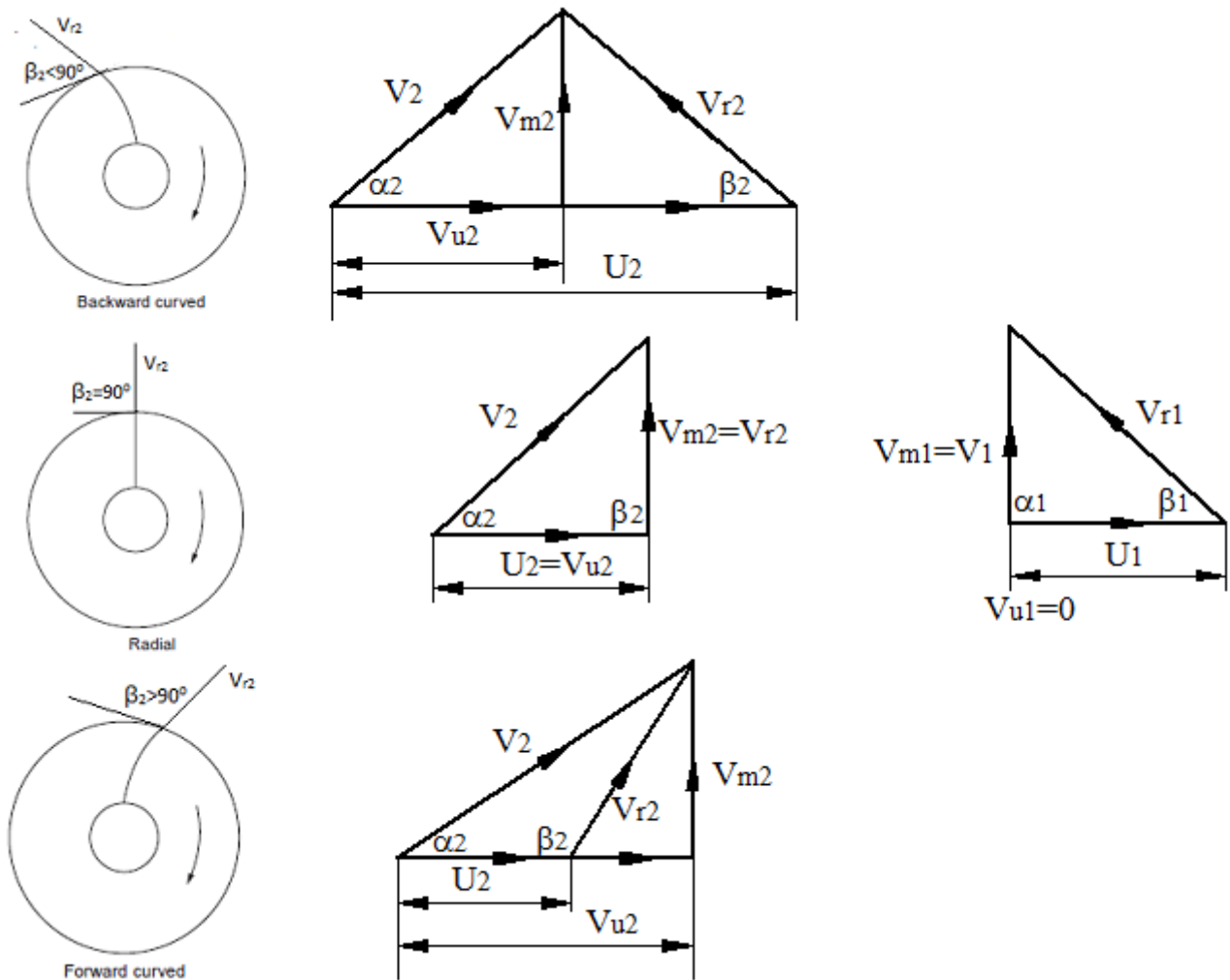


Fig. 7.2 Different vane shapes in centrifugal compressors.

**7.4 Influence of Impeller Vane Shape:**

**Question No 7.4:** Discuss with velocity diagram why backward curved vanes are preferred for radial flow (centrifugal) compressors.

**Answer:** Figure 7.2 gives the velocity diagram for different vane shapes. For compressor which absorb a specific amount of energy and runs at a given speed, the diameter ( $D_2$ ) and whirl velocity ( $V_{u2}$ ) can be varied to maintain  $U_2 V_{u2}$  at the required value. Hence if  $\beta_2$  is large (as in forward curved vane),  $V_{u2}$  is also large and  $U_2$  has to be small that is diameter should be decreased. Similarly, if  $\beta_2$  is small (as in backward curved vane),  $V_{u2}$  is small. Hence  $U_2$  has to be large and the diameter should be increased appropriately to provide the required performance. This implies that compressor with backward curved vanes are larger in size than those with forward curved (or radial) vanes of the same capacity.

Now consider fluid flow through compressors running at the same tip speed ( $U_2$ ) and with the same radial velocity ( $V_{m2}$ ). Then, an increase in exit vane angle ( $\beta_2$ ) increases exit fluid velocity ( $V_2$ ) consequently, a very efficient diffuser is needed to obtain a pressure rise using all to of the kinetic energy at the exit. Because of irreversibilities due to adverse pressure gradients and thick boundary



layers, complete diffusion of the exit kinetic energy with a pressure rise corresponding to the theoretical, is impossible. Therefore the compressors with large exit angles will be less efficient overall than compressors with small exit angles. So when high compressor efficiency is desired, machines with backward curved vanes must be used. This is one of the reasons that compressors with backward curved vanes are preferred. In some cases, where a large pressure rise is needed with a compressor of small size, radial blades are used though the efficiency may not be as high as that of a compressor with backward curved vanes of similar capacity. Compressors with forward curved vanes are even less common than those of radial type.

### 7.5 Slip and Slip Factor:

**Question No 7.5: Briefly explain the slip and slip coefficient in centrifugal compressors.**

(VTU, Jun-12)

**Answer:**

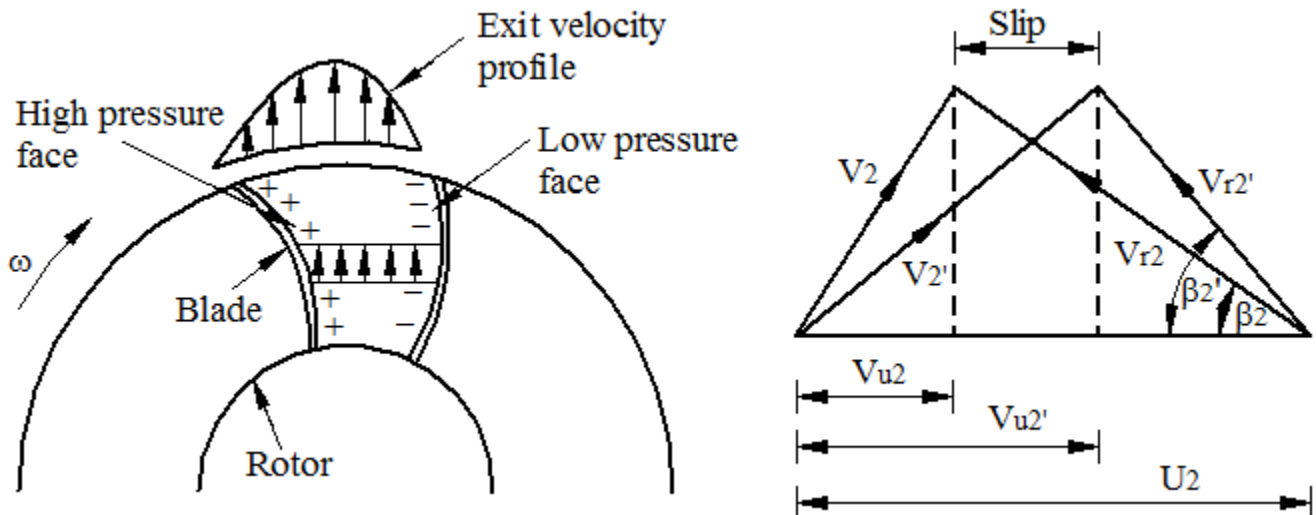


Fig. 7.3 Velocity and pressure distribution over impeller

In deriving the Euler's equation, it was assumed that the velocity and pressure distributions are uniform over the impeller cross-sectional area. But in actual practice this assumption is not correct, because the velocity and pressure are not uniform over an impeller cross-sectional area as shown in figure 7.3. Due to uneven pressure distribution and hence the velocity distribution the tangential component of the velocity (whirl velocity) reduces, thus head developed by the machine is always less than that developed at the ideal condition.

Thus slip may be defined as the phenomenon observed in centrifugal machines due to uneven pressure distribution and the velocity distribution, which results in the reduction of tangential component of the velocity (whirl velocity).

If  $V_{u2'}$  is the tangential component of the velocity without slip and ( $V_{u2}$ ) is the tangential component of the velocity with slip, then slip ( $S$ ) is  $S = V_{u2'} - V_{u2}$

The ratio of ideal head ( $H_i$ ) with slip to the Euler's head ( $H_e$ ) without slip is called the slip coefficient ( $\mu$ ).

$$\mu = \frac{H_i}{H_e} = \frac{V_{u2}}{V_{u2'}}$$

Then theoretical work done on gas by a centrifugal compressor with slip is given as:

$$w_{the} = \Delta h_o = \mu U_2 V_{u2'} = \mu e$$

Although above equation modified by the slip coefficient, it is still the theoretical work done on the gas, since slip will be present even if the fluid is frictionless (ideal fluid).

### 7.6 Power Input Factor and Pressure Coefficient:

The losses that occur in a compressor are due to:

- (i) Friction between air and the sides of the passages of flow or between disks.
- (ii) The effects of shock (due to improper incidence), separation in regions of high adverse pressure gradients and turbulence.
- (iii) Leakage between the tip of the rotor and casing.
- (iv) Mechanical losses in bearings etc.

The frictional losses in the rotor and leakage losses between the tip of the rotor and casing make the actual work absorbed by the rotor, lesser than the theoretical. This fact is expressed by a quantity called the power input factor or the work factor, which is defined as the ratio of actual work supplied to the compressor to the theoretical work supplied to the same machine.

$$\Omega = \frac{w}{w_{the}} = \frac{w}{\mu U_2 V_{u2'}}$$

Then actual work supplied to the compressor is,

$$w = \Omega \mu U_2 V_{u2'} = \Omega \mu e$$

The frictional losses in the diffuser and the loss due to exit kinetic energy make the total static pressure rise, less than the theoretical maximum specified by the impeller speed. This fact is expressed by a quantity called the pressure or loading coefficient, which is defined as the ratio of the isentropic work needed to cause the observed rise to the isentropic work specified by the impeller tip-speed (Euler's work).

$$\psi_p = \frac{w_{ise}}{e}$$

But,

$$\text{isentropic work } (w_{ise}) = \text{actual work} \times \text{isentropic efficiency} = w \times \eta_c$$

$$w_{ise} = \eta_c \Omega \mu e$$

Then pressure coefficient is,

$$\psi_p = \frac{w \times \eta_c}{e} = \frac{\eta_c \Omega \mu U_2 V_{u2'}}{U_2 V_{u2'}}$$

$$\psi_p = \eta_c \Omega \mu$$

### 7.7 Work Done and Pressure Ratio:

**Question No 7.6:** Derive an expression for total-to-total pressure ratio in terms of impeller tip speed for a radial vanes centrifugal compressor. (VTU (Ph.D), Jan-12)

**Answer:** Figure 7.4 shows the enthalpy-entropy diagram for a centrifugal compressor stage. State points with superscript are correspond to isentropic compression processes. Air enters the impeller vanes with lower absolute velocity  $V_1$  and leaves with large absolute velocity  $V_2$ . This absolute velocity reduces to  $V_3$  when it passes through the diffuser vanes, which is slightly higher than  $V_1$ . It can be observed that stagnation pressure  $P_{o1}$  will be higher than static pressure  $P_1$  by an amount  $\frac{V_1^2}{2}$ . Similarly  $P_{o2}$  is much higher than static pressure  $P_2$  by an amount  $\frac{V_2^2}{2}$ .

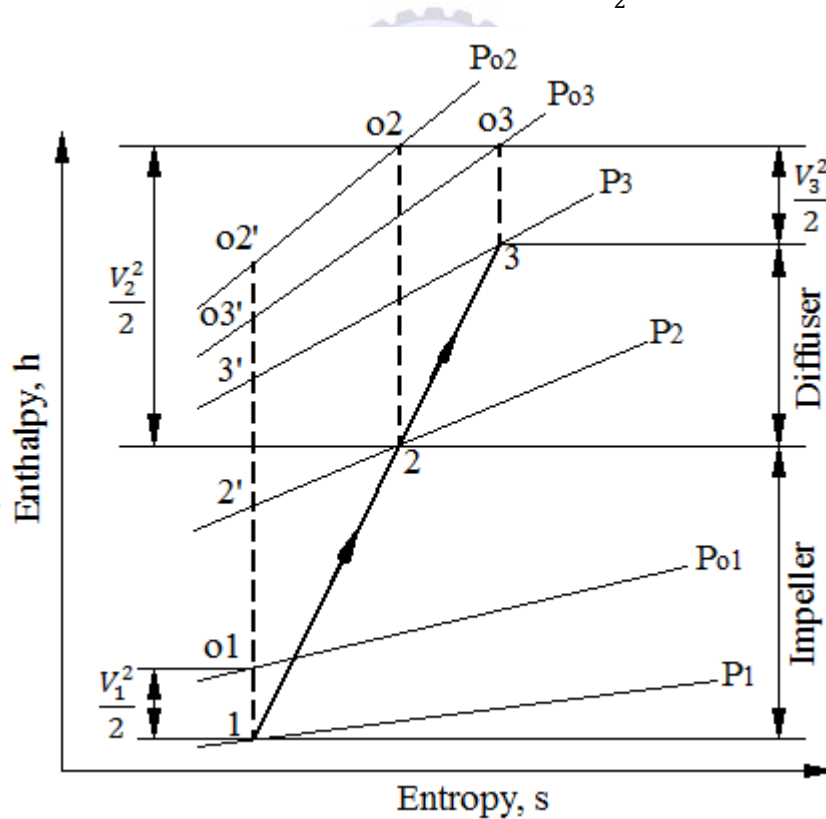


Fig. 7.4 Enthalpy-entropy diagram of a centrifugal compressor

Euler’s work done on gas by a centrifugal compressor is given as:

$$e = U_2 V_{u2'}$$

(Because,  $V_{u1} = 0$ )

For backward curved vanes,

$$V_{u2'} = U_2 - V_{m2} \cot \beta_2$$

Then, 
$$e = U_2(U_2 - V_{m2} \cot \beta_2)$$

For forward curved vanes,

$$V_{u2'} = U_2 + V_{m2} \cot(180^\circ - \beta_2) = U_2 - V_{m2} \cot \beta_2$$

Then, 
$$e = U_2(U_2 - V_{m2} \cot \beta_2)$$

For radial vanes,

$$V_{u2'} = U_2$$

Then, 
$$e = U_2^2$$

The stage efficiency of the compressor based on stagnation conditions at entry and exit is given by,

$$\eta_c = \frac{\text{Total isentropic enthalpy rise between inlet and outlet}}{\text{Actual enthalpy rise between same total pressure limits}}$$

$$\eta_c = \frac{h_{o3'} - h_{o1}}{h_{o3} - h_{o1}} = \frac{c_p(T_{o3'} - T_{o1})}{h_{o3} - h_{o1}}$$

Since, no work is done in the diffuser,  $h_{o2} = h_{o3}$ .

$$h_{o3} - h_{o1} = h_{o2} - h_{o1} = w = \Omega \mu U_2 V_{u2'}$$

$$\eta_c = \frac{c_p T_{o1} \left( \frac{T_{o3'}}{T_{o1}} - 1 \right)}{\Omega \mu U_2 V_{u2'}} = \frac{c_p T_{o1} \left[ \left( \frac{P_{o3}}{P_{o1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\Omega \mu U_2 V_{u2'}}$$

Then pressure ratio of centrifugal compressor is,

$$P_{ro} = \frac{P_{o3}}{P_{o1}} = \left[ 1 + \frac{\eta_c \Omega \mu U_2 V_{u2'}}{c_p T_{o1}} \right]^{\frac{\gamma}{\gamma-1}}$$

For backward curved vanes,

$$P_{ro} = \left[ 1 + \frac{\eta_c \Omega \mu U_2 (U_2 - V_{m2} \cot \beta_2)}{c_p T_{o1}} \right]^{\frac{\gamma}{\gamma-1}}$$

For forward curved vanes,

$$P_{ro} = \left[ 1 + \frac{\eta_c \Omega \mu U_2 (U_2 + V_{m2} \cot \beta_2)}{c_p T_{o1}} \right]^{\frac{\gamma}{\gamma-1}}$$

For radial vanes,

$$P_{ro} = \left[ 1 + \frac{\eta_c \Omega \mu U_2^2}{c_p T_{o1}} \right]^{\frac{\gamma}{\gamma-1}}$$

The interesting part of the above equation is that they permit direct evaluations of pressure ratio and work output, once the initial conditions and the rotor tip-speed are given and slip-coefficient, power input factor and efficiency are estimated.

## 7.8 Compressibility and Pre-whirl:

**Question No 7.7: Discuss the compressibility for a centrifugal compressor. (VTU, Dec-12)**

**Answer:** The Mach number is responsible for the compressibility of a flow. The higher the Mach number the greater will be the compressibility effect and hence it reduces the compressor efficiency. The Mach number at the inlet of the impeller eye is mainly depending on the relative velocity of the impeller at the inlet. It is therefore necessary to keep the relative velocity value as low as possible.

For a given flow rate, impeller eye may be either large or small. For large eye, velocity  $V_1$  is low and eye tip speed  $U_1$  is high. For small eye it is opposite. Both these conditions result in higher value of  $V_{r1}$ , but it is minimum in between these two. Figure 7.5 shows the variation of relative Mach number with the eye tip diameter.

**Question No 7.8: What is the necessity of providing the pre-whirl at the inlet of the centrifugal compressor? (VTU, Dec-11)**

**Answer:** When the relative velocity is too high for efficient operation of a compressor and if flow rate and speed cannot be altered, still the relative velocity can be reduced by giving the fluid some initial positive pre-rotation. This is known as *Prewirl*. This is usually done by providing inlet guide vanes installed directly in front of the eye as shown in figure 7.6. Sufficient prewhirl at the eye tip can avoid reaching the condition of critical Mach number leading to a prewhirl component. Positive prewhirl is disadvantageous because a positive inlet whirl velocity reduces energy transfer by an amount equal to  $U_1 V_{u1}$ .

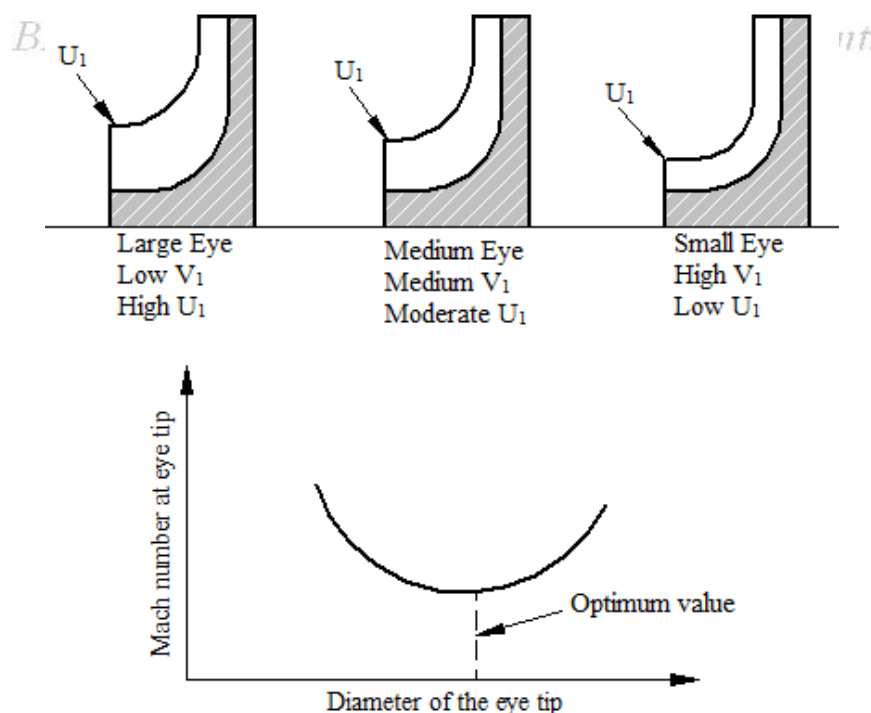


Fig. 7.5 Variation of Mach number with the eye tip diameter

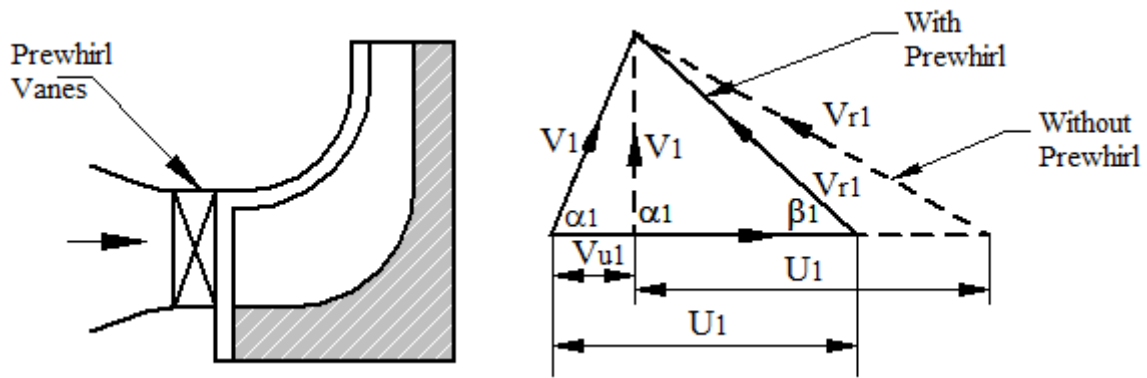


Fig. 7.6 Prewirl at impeller eye

Note: **Compressibility** is a measure of the relative volume change of a fluid or solid as a response to a pressure change.

**7.9 Diffuser:**

**Question No 7.9:** What is the function of a diffuser? Name different types of diffusers used in centrifugal compressor and explain them with simple sketches. (VTU, Jun/Jul-09)

**Answer:** Diffuser plays an important role in the overall compression process of a centrifugal compressor. The impeller imparts energy to the air by increasing its velocity. The diffuser converts this imported kinetic energy into pressure rise. For a radial vanned impeller, the diffuser does compress and increase the pressure equal to 50 percent of the overall static pressure rise.

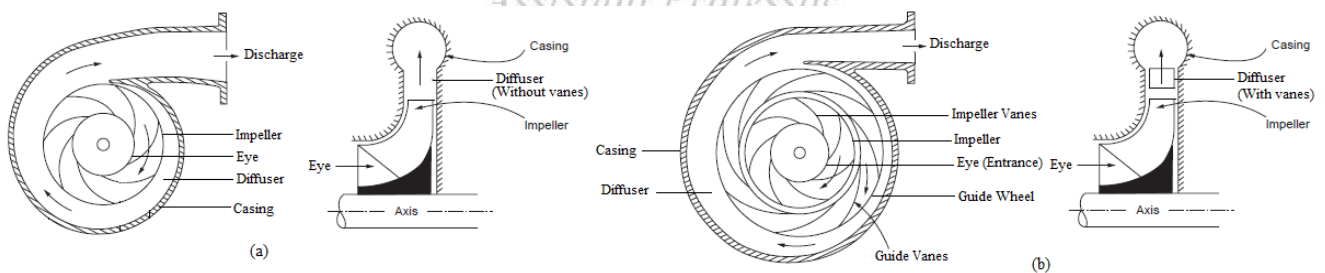


Fig. 7.7 Types of diffusers

**7.9.1 Vaneless Diffuser:** In this type, the diffusion process will take place in the vaneless space around the impeller before the air leaves the compressor stage through volute casing. A vaneless diffuser is shown in figure 7.7 (a). A vaneless diffuser has wide range of mass flow rate. But for a large reduction in the outlet kinetic energy, diffuser with a large radius is required. Because of long flow path with this type of diffuser, friction effects are important and the efficiency is low.

**7.9.2 Vanned Diffuser:** In the vanned diffuser as shown in figure 7.7 (b), the vanes are used to diffuse the outlet kinetic energy at a much higher rate, in a shorter length and with a higher efficiency than the

vaneless diffuser. A ring of diffuser vanes surrounds the impeller at the outlet, and after leaving the impeller, the air enters the diffuser vanes

The diffuser efficiency defined as the ratio of ideal enthalpy rise to the actual enthalpy rise in the diffuser.

$$\eta_d = \frac{\text{Ideal enthalpy rise}}{\text{Actual enthalpy rise}}$$

### 7.10 Centrifugal Compressor Characteristics:

An idealized centrifugal compressor characteristic curve is shown in figure 7.8. Consider a centrifugal compressor delivering through a flow control valve situated after the diffuser. There is a certain pressure head, even if the valve is fully closed and is indicated by state 1. This pressure head is merely due to the churning action of the impeller vanes. The pressure head so developed is called “shut off” head. As flow control valve is opened, the air starts flowing and diffuser increases the pressure head. Thus, at state 2, the maximum pressure head is reached but the efficiency is just below the maximum efficiency.

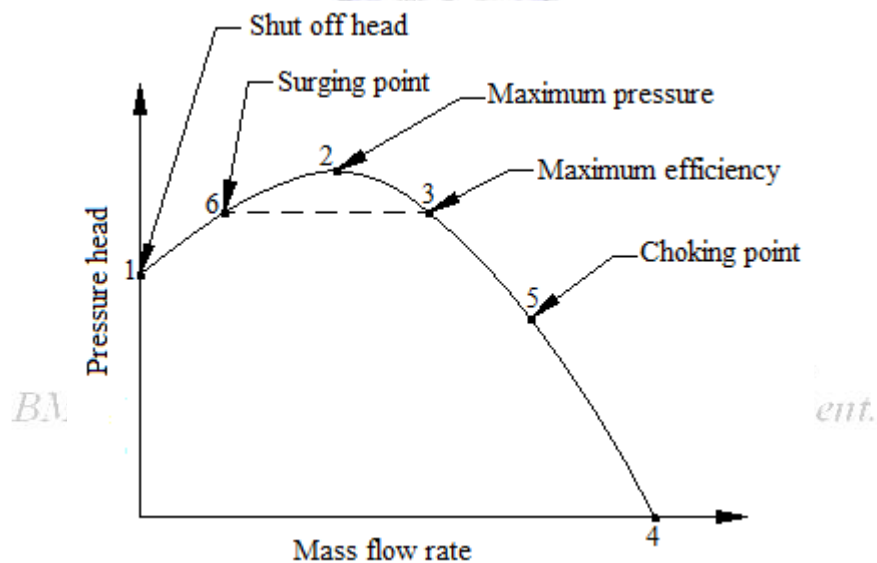


Fig. 7.8 Characteristic curve of centrifugal compressor

Further increase in mass flow reduces the pressure head to state 3. But at this state, the efficiency is maximum compared with state 2. Thus the value corresponding to state 3 is said to be design mass flow rate and pressure head.

Further increase in mass flow decreases the pressure head and reaches zero at state 4. Corresponding to this state, all the power absorbed by the compressor is used to overcome the internal friction and thus the compressor efficiency is zero.

#### 7.10.1 Surging:

**Question No 7.10:** Explain the surging phenomena in centrifugal compressors with the help of head-discharge curves. (VTU, Dec-08/Jan-09, Dec-11, Jun-12, Jun/Jul-13)

**Answer:** The phenomenon of a momentary increase in the delivery pressure resulting in unsteady, periodic and reversal of flow through the compressor is called surging.

Consider compressor is operating at the state 3 as shown in figure 7.8, if the mass flow is reduced by gradual closing of the flow valve, the operating point move on to the left. Further reduction in mass flow increases the pressure head until it reaches the maximum value. Any further decrease in flow will not increase the pressure head and hence reduces the pressure head to state 6. At this condition there is a large pressure in the exit pipe than at compressor delivery and the flow stops momentarily, and may even flow in the reverse direction. This reduces the exit pipe pressure, then compressor again starts to deliver the air and the operating point quickly shifts to 3 again. Once again the pressure starts increasing and operating point moves from right to left. If the exit pipe conditions are remain unchanged then once again the flow will breakdown after state 2 and cycle will be repeated with high frequency. This phenomenon is called surging.

If the surging is severe enough then the compressor may be subjected to impact loads and high frequency vibration leads to failure of the compressor parts.

#### 7.10.2 Choking:

**Question No 7.11:** Explain the choking phenomena in centrifugal compressors. (VTU, Dec-11,)

**Answer:** When the mass flow is increased to the right of point 3 on the characteristic curve (as in figure 7.8) a state 5 is reached, where no further increase in mass flow is possible no matter how wide open the flow control valve is. This indicates that the flow velocity in the passage reaches the speed of sound at some point within the compressor and the flow chokes. Choking means fixed mass flow rate regardless of pressure ratio. Choking may take place at the inlet, within the impeller, or in the diffuser section. It will occur in the inlet if stationary guide vanes are fitted.